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ENGINEERING DESIGN HANDBOOK

TRAJECTORIES, DIFFERENTIAL EFFECTS, AND DATA FOR PROJECTILES

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SYMBOLS

A—Azimuth, measured from the direction of motion of the airplane (in aircraft fire)

A—Altitude function (Eq. 63)

A—Axial moment of inertia

B—Drag coefficient in lb/in^2-ft (Eq. 19)

C—Ballistic coefficient (Eq. 22)

D—Drag

D—Angular deflection

F—Temperature in degrees Fahrenheit

G—Drag function (Eq. 20)

H—Ratio of air density at altitude y to that at the surface (Eq. 48)

H—Height of target

I—Inclination function (Eq. 62)

K—Coefficient (See Eq. 16 and 34)

L—Crosswind force (lift)

M—Mach number (Eq. 18)

$\overline{M}$—Pseudo-Mach number (Eq. 65)

M—Overturning moment about the center of gravity

M—$\log_{10}e = 0.43429$ (approx.)

N—Spin

P—Performance parameter (Eq. 27)

P—Siacci range: the distance measured along the line of departure to a point directly above the projectile (Eq. 54)

Q—Drop of the projectile (Eq. 55)

R—Reynolds number (Eq. 17)

R—Range

S—Space function (Eq. 60)

T—Total time of flight

T—Time function (Eq. 61)

U—Upper limit of integration of Siacci functions

V—Velocity

W—Wind speed

X—Total range of a trajectory

Y—Altitude of an airplane above sea level

Z—Linear deflection

Z—Zenith angle

a—Velocity of sound

Diameter of the largest cross section of the projectile (in Gâvre drag law, Eq. 23)

c′—Damping coefficient due to yawing moment and crosswind force (Eq. 51)

c″—Damping coefficient due to drag (Eq. 33)

d—Caliber, or maximum diameter of projectile

f—Drag coefficient (in the Gâvre drag law, Eq. 23)

g—Gravitational acceleration

h—Logarithmic rate of decrease of air density with altitude: 0.000,031,58 per ft

h—Yawing moment damping factor

i—Form factor (Eq. 21)

k—Logarithmic rate of decrease of sonic velocity with altitude 0.000,003,01 per ft

k—Ratio of American to French drag function (Eq. 24)

m—Mass of the projectile

m—Slope of the tangent to the trajectory

n—Logarithmic rate of change of muzzle velocity with projectile weight

p—Space function (Eq. 28)

p—Projectile weight

q_1—Inclination function (Eq. 30)

q—Altitude function (Eq. 31)

s—Stability factor

t—Time of flight (sometimes the symbol t_f is used to distinguish the time of flight from the Siacci time function)

t—Time function (Eq. 29)

u—Velocity of the projectile relative to the air

u—Component along the line of departure of the velocity relative to the air: the argument of the Siacci functions

v—Velocity of the projectile relative to an inertial system

w—True air speed of the airplane in aircraft fire

x—Horizontal distance from the origin of the trajectory

y—Vertical distance from the origin of the trajectory

z—Drift

Δ (capital delta) or Δ'—Increment of (followed by a variable)

Δ—Density of air (in Gâvre drag law, Eq. 23)

γ (gamma)—Factor inversely proportional to the ballistic coefficient (Eq. 66)

δ (delta)—Yaw

ε (epsilon)—Angle of site (Eq. 72)

ϑ (theta)—Angle of inclination of the trajectory

$\varkappa$ (kappa)—Crosswind force damping factor

μ (mu)—Viscosity of the air

ρ (rho)—Density of the air

τ (tau)—Temperature of the air

φ (phi)—Angle of departure

ω (omega)—Angle of fall

Superscripts

One dot (example: $\dot{x}$)—Derivative with respect to time.

Two dots (example: $\ddot{y}$)—Second derivative with respect to time.

Subscripts

0—At $t = 0$, $y = 0$, or $\delta = 0$.

1—Pertaining to, or relative to, Projectile Type 1

2—Pertaining to, or relative to, Projectile Type 2

8—Pertaining to, or relative to, Projectile Type 8

.1—First revision

.2—Second revision

C—Due to 1% increase in ballistic coefficient

D—Drag

H—Due to height of target

R—Range component of

e—Effective

p—Due to projectile weight

s—Summital: at the summit of a trajectory, where $\dot{y} = 0$

s—Standard: pertaining to standard conditions

t—Pertaining to, or relative to, some Typical Projectile

v—Due to 1 unit increase in muzzle velocity

w—Due to wind

z—Cross component of

δ—Yaw

τ—Due to increase in the temperature of the atmosphere

φ—Due to an increase in angle of departure

ω—Terminal: where $y = 0$ on the descending branch of the trajectory

PREFACE

This pamphlet, one of the series which will comprise the Ordnance Engineering Design Handbook, deals with the subject of Exterior Ballistics, covering Trajectories, Differential Effects and Data for Projectiles. The pamphlet pertains to that phase of projectile life starting at the gun muzzle (or, for a rocket, at burnout) and ending at the point of burst or impact. The subject is covered in a manner intended primarily to assist the ammunition designer in recognizing and dealing with the design parameters of projectile weight, shape, muzzle velocity, yaw, drag and stability, in designing for optimum results in range, time of flight and accuracy. It is hoped that it will also prove helpful to other personnel of the Ordnance Corps and of contractors to the Ordnance Corps. Because of its complexity, it is possible to cover the subject only briefly in this volume, but an attempt has been made to incorporate references which will permit the designer or the student to explore in more detail any phase of the subject.

The ballistics of bombs and of intermediate and long range missiles are not covered in this pamphlet. Neither are the problems associated with Interior Ballistics or of Terminal Ballistics. These subjects will be covered by other pamphlets.

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TRAJECTORIES, DIFFERENTIAL EFFECTS, AND DATA FOR PROJECTILES

1. Introduction to Exterior Ballistics

Ballistics is the science of the motion and effects of projectiles. Exterior Ballistics deals with that part of the motion between the projector and the point of burst or impact.

The projector may be any sort of weapon: a sling-shot, a bow, a mortar, a rifle, a rocket launcher, an airplane, etc. A projectile may be any body that is projected by such a weapon. However, in this pamphlet, the term "projectile" will be restricted to those missiles that are in free flight through the air after being propelled by a gun or a rocket motor. This term does not include bombs, which are released from airplanes; they will be considered in another pamphlet.

The motion of a missile in flight is really quite complicated. Even a fin-stabilized projectile or bomb that is not spinning about its longitudinal axis is swinging about a transverse axis, and this affects its motion. The nose of a spinning shell precesses about the tangent to its trajectory, and its attitude gives rise to forces that not only affect its motion in a vertical plane but also move it out of this plane. To simplify the calculations, however, it is usually assumed that a missile acts like a particle subject to two forces: (1) gravity, which is directed vertically downward, and (2) drag, which is directed opposite to the direction of motion. Gravity is assumed constant: until 1956, the gravitational acceleration was usually taken to be 32.152 ft/sec^2 (9.80 m/sec^2); and was used for all tables referred to in this text; now, the standard value is 32.174 ft/sec^2 (9.80665 m/sec^2).[1] The drag depends on the size of the missile, its velocity, the density of air, and other factors. Under these conditions, the motion follows a smooth, planar path, which is called an "exterior ballistic trajectory," or "trajectory" for short.

The initial conditions for the trajectory of a particular missile are its initial velocity and angle of departure.* For a projectile, the initial velocity is the muzzle velocity and angle of departure is approximately the elevation of the weapon (fictitious values of these quantities may be used in the case of a rocket that burns outside the launcher). A bomb is released with the speed of the airplane, generally in a horizontal direction. If a gun is mounted in an airplane or other moving vehicle, the initial conditions are affected by the motion of the vehicle. In any case, it is customary to compute trajectories for a few selected values of initial velocity and elevation.

Certain standard conditions are specified for a normal trajectory: not only the initial velocity and angle of departure, but also the weight of the projectile and a factor that depends on its shape; the density, temperature, and lack of motion of the air, and the height of the target relative to the projector. In order to determine the effects of variations from these conditions, it may be convenient in some cases to compute non-standard trajectories; however, they can usually be estimated in other ways, as will be explained later.

2. Vacuum Trajectories

a. Equations of Motion

A rough idea of an exterior ballistic trajectory may be gained by neglecting the air resistance, considering only the effects of gravity. Then if

- x is the horizontal distance from the origin,
- y is the vertical distance from the origin,
- t is the time of flight,
- g is the gravitational acceleration,

and dots denote derivatives with respect to time, the differential equations of motion are

$$\ddot{x} = 0, \; \ddot{y} = -g. \tag{1}$$

The initial conditions, denoted by the subscript 0, are

$$\dot{x}_0 = v_0 \cos \vartheta_0, \dot{y}_0 = v_0 \sin \vartheta_0, x_0 = 0, y_0 = 0,$$

where v is the velocity,

ϑ the angle of inclination.

By integrating equations (1), we find that, at any time,

$$\dot{x} = v_0 \cos \vartheta_0, \;\; \dot{y} = v_0 \sin \vartheta_0 - gt, \tag{2}$$

and by integrating (2),

* The term ***angle of departure*** and the term ***elevation*** as used in this pamphlet represent the angle from the ***horizontal*** to the line of departure and therefore would be more accurately and completely termed ***quadrant angle of departure.***

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and by integrating (2),

$$x = (v_o \cos \vartheta_o)t,\ y = (v_o \sin \vartheta_o)t - gt^2/2. \quad (3)$$

b. Characteristics of Trajectories

From Equations (3), we find that the time at any horizontal range is

$$t = x/v_o \cos \vartheta_o \quad (4)$$

and hence the ordinate is

$$y = x \tan \vartheta_o - gx^2/2v_o^2 \cos^2 \vartheta_o. \quad (5)$$

This is a parabola with a vertical axis.

The total range X is found by letting $y = 0$ in (5) and solving for x:

$$X = (2v_o^2/g) \sin \vartheta_o \cos \vartheta_o = (v_o^2/g) \sin 2\vartheta_o. \quad (6)$$

This shows that the range is a maximum when $\vartheta_o = 45°$.

The total time of flight T is found by substituting (6) for x in (4):

$$T = (2v_o/g) \sin \vartheta_o. \quad (7)$$

The components of the terminal velocity are found by substituting (7) for t in (2):

$$\dot{X} = v_o \cos \vartheta_o,\ \dot{Y} = -v_o \sin \vartheta_o. \quad (8)$$

Hence the terminal velocity is

$$v_\omega = v_o, \quad (9)$$

and the angle of fall (considered positive) is

$$\omega = \vartheta_o. \quad (10)$$

By differentiating equation (5) with respect to x, we find that the slope of the trajectory at any point is

$$\frac{dy}{dx} = \tan \vartheta_o - gx/v_o^2 \cos^2 \vartheta_o. \quad (11)$$

At the summit, the slope is 0; hence the horizontal range to the summit is

$$x_s = (v_o^2/g) \sin \vartheta_o \cos \vartheta_o = X/2. \quad (12)$$

By substituting this for x in (5), we find the maximum ordinate

$$y_s = (v_o^2/2g) \sin^2 \vartheta_o = (1 - \cos 2\vartheta_o)\, v_o^2/4g$$
$$= (g/8)\, T^2. \quad (13)$$

By substituting (12) for x in (4), we find the time to the summit

$$t_s = (v_o/g) \sin \vartheta_o = T/2. \quad (14)$$

3. Air Resistance

a. Forces Acting on a Missile Moving in Air

A missile moving in air encounters other forces besides gravity. The principal components of the force produced by motion through air are drag, crosswind force, and, for a spinning projectile, Magnus force.

(1) *Drag* is the component in the direction opposite to that of the motion of the center of gravity. This is the only component acting on a non-spinning projectile that is trailing perfectly.

(2) *Crosswind Force* in the component perpendicular to the direction of motion of the center of gravity in the plane of yaw. The yaw is the angle between the direction of motion of the center of gravity and the longitudinal axis of the missile. This component is the cause of the drift of a spinning shell.

(3) *Magnus Force* acts in a direction perpendicular to the plane of yaw. This is the force that makes a spinning golf ball or baseball swerve.

b. Simplified Assumptions

The motion caused by these forces is explained in detail in the Ordnance Corps Pamphlet, "Design for Control of Flight Characteristics."[2] In general, the motion of the center of gravity is affected by the motion about the center of gravity and occurs in three dimensions. To simplify the calculations, it is usually assumed that the only forces acting on the missile in free flight are drag and gravity. The crosswind force is implied in the drift, but this is determined experimentally as a variation from the plane trajectory. This approximation is satisfactory except at extremely high elevations, where the summital yaw is very large.

c. *Drag Coefficient*

The *drag* encountered by a projectile moving in air is defined by the equation of motion

$$m \frac{dv}{dt} = -D - mg \sin \vartheta, \quad (15)$$

where

- m—is the mass of the projectile,
- v—the velocity of the projectile relative to an inertial system,
- t—the time,
- D—the drag,
- g—the gravitational acceleration,
- ϑ—the angle of inclination.

The *drag coefficient* is defined by the relation

$$K_D = D/\rho d^2 u^2, \quad (16)$$

where

- K_D—is the drag coefficient,
- ρ—the air density,
- d—the caliber or maximum diameter,
- u—the velocity of the projectile relative to the air.

The drag coefficient is a function of the Reynolds

$$x = (v_o \cos \vartheta_o)t, \; y = (v_o \sin \vartheta_o)t - gt^2/2. \quad (3)$$

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where

K_D—is the drag coefficient,
ρ—the air density,
d—the caliber or maximum diameter,
u—the velocity of the projectile relative to the air.

The drag coefficient is a function of the Reynolds

number R and the Mach number M. The Reynolds number is defined as

$$R = ud\rho/\mu \qquad (17)$$

where μ is the viscosity of the air, and the Mach number as

$$M = u/a, \qquad (18)$$

where a is the velocity of sound in the surrounding medium.* The viscosity is related to the friction of the air; the velocity of sound, to its elasticity. At extremely low velocities, the Reynolds number causes greater variation, but the effect of the Mach number is larger when it exceeds 0.5. The angle between the direction of motion of the center of gravity and the axis of symmetry of the shell, called yaw, also affects the drag coefficient, but this is usually stripped out of the resistance firing data.

Figure 2 (p. 22) is a graph of drag coefficient vs Mach number for the four typical projectiles whose shapes are shown in Figure 1 (p. 21). These projectiles are described and the basic firings explained in Section 4. It should be noted that subscripts are used to distinguish the typical projectiles to which the drag coefficients pertain; a second subscript is sometimes added after a decimal point to denote a revision. It is evident that all the drag coefficients increase rapidly in the vicinity of Mach number 1. The main difference between curves is in the ratio of the maximum, which occurs between Mach numbers 1.0 and 1.5, to the minimum subsonic value. Although the drag coefficient has been determined and tabulated for several other projectiles, these four represent the principal shapes for artillery projectiles.

K_D is a non-dimensional coefficient, and a consistent set of units should be used in Equation (16). However, it is customary to express the density as a ratio, the caliber in inches, and the velocity in feet per second; then, since the drag is in poundals, the drag coefficient is expressed in pounds per square inch per foot and is denoted by B. Until 1956, the standard air density was taken as

$$\rho_o = 0.07513 \text{ lb/ft}^3 = 5.217 \times 10^{-4} \text{ lb/in}^2\text{-ft.*}$$

The practical drag coefficient,

$$B = 5.217 \times 10^{-4} K_D, \qquad (19)$$

for Projectile Types 1, 2 and 8 is tabulated as a function of velocity for the standard velocity of sound,

$$a_o = 1120.27 \text{ fps.*}$$

The non-dimensional drag coefficient K_D for the 90-mm HEAT Shell T108 is tabulated as a function of Mach number. (See References 3, 4, 5 and 6, which may be obtained from the Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland).

d. Drag Function

The drag function G is computed by the formula

$$G = Bu \qquad (20)$$

with u in feet per second. For convenience in computing trajectories, this has been tabulated as a function of $u^2/100$ with u in meters per second and either feet or yards per second, for Projectile Types 1, 2 and 8. (See References 7, 8 and 9.)

e. Form Factor

It is not practical to determine the drag coefficient as a function of Mach number for all projectiles. Instead, the drag coefficient of a projectile whose shape differs only slightly from one of the typical projectiles is assumed to be proportional to the typical drag coefficient. If K_{D_t} is the drag coefficient of a Typical Projectile, the form factor of a projectile whose drag coefficient is K_D is the ratio

$$i_t = K_D/K_{D_t}. \qquad (21)$$

The form factor may be determined from the time of flight at short ranges, or from the range to impact or the position of burst at a few elevations. If it is desired to estimate the form factor of a projectile without firing it, the Ordnance Corps Pamphlet, "Design for Control of Flight Characteristics"[2] should be consulted. It explains the effects on drag of variations in length and radius of head, méplat diameter, base area, length of shell, and yaw.

Generally, the form factor of a projectile with a square base and an ogive less than 1.75 calibers high should be referred to Projectile Type 1; that of a projectile with a square base and an ogive more than 1.75 calibers high, to Projectile Type 8; that of a projectile with a boattail, to Projectile Type 2; and that of a finned projectile, to the T108 Shell. However, there are exceptions to these rules: e.g., K_{D_1} is used for fin-stabilized mortar shells at low velocities, and K_{D_2} fits the calculated drag coefficient for some longer fin-stabilized shells. The drag coefficients of bombs are treated elsewhere.[10]

* In 1956, the Department of Defense adopted the standard atmosphere of the International Civil Aviation Organization.[1] In this atmosphere, at the surface, the density is 0.076,475 lb/ft³ and the velocity of sound is 1116.89 fps; also the viscosity is 1.205 x 10^{-5} lb/ft-sec, but the effect of Reynolds number is not considered in computing trajectories.

f. Ballistic Coefficient

The ballistic coefficient relative to the tabulated drag coefficient is defined by the formula

$$C_t = m/i_t d^2, \qquad (22)$$

where m is the mass of the projectile in pounds,
d is the caliber (or maximum diameter) in inches.

g. Air Density

The air density is a function of altitude. The exponential function adopted by the Ordnance Corps* is tabulated in References 7 and 11. The ratio of the density at an altitude y to that at the surface is H(y).

4. Description of Typical Projectiles

This pamphlet contains data that pertain specifically to four types of projectiles: Types 1, 2 and 8 and the 90-mm HEAT Shell T108. They can be applied to other shapes by using a form factor, as explained in paragraph 3e.

a. Projectile Type 1

Most of the service projectiles used by the European artillery during the last third of the nineteenth century had square bases and ogival heads of various apical angles. In Russia, during the years 1868 and 1869, General Magenski conducted some experiments at St. Petersburg to determine the law of air resistance of projectiles with a semi-apical angle of 48°22′ (1.49-caliber radius head) at velocities from 172 to 409 m/s. In England, from 1866 to 1880, Bashforth conducted similar experiments with projectiles of the same form (it may be that the tables were reduced to this form) at velocities from 148 to 825 m/s. In Holland, Colonel Hojel conducted a few such experiments during 1883; the exact form is uncertain, but the ogive was evidently somewhat longer than those of the Russian experiments. In Germany, the Krupp factory conducted a large number of air resistance experiments at Meppen Proving Ground, starting in 1881 (the principal results were reported by Siacci in the "Rivista" for March 1896); the velocities varied from 368 to 910 m/s, and three different types of projectiles were used: one was the same as the Russian (or the results reduced to the same), but the others probably had longer ogives. In France, a commission of the naval artillery conducted many such experiments at Gâvre Proving Ground from 1873 to 1898 at velocities from 118 to 1163 m/s; they fired ogival projectiles with three semi-apical angles: 45°55′ (1.64 CRH), 41°40′ (1.98 CRH), and 31°45′ (3.34 CRH). For further details, see References 12 to 16.

In analyzing the results of the experiments,[17] the Gâvre Commission of Experiments assumed that the drag could be expressed in the form

$$D = \frac{\Delta}{g} a^2 V^2 f(V), \qquad (23)$$

where

D—is the drag (kg),
Δ—the density of the air (kg/m^3),
g—the gravitational acceleration (m/sec^2),
a—the diameter of the largest cross section of the projectile (m),
V—the velocity (m/s).

The drag was treated as a function of velocity without taking account of the velocity of sound, although the dependence of drag on the velocity of sound was strongly suspected: the coefficient f was plotted as a function of V alone. After studying the results obtained with projectiles of various ogival angles, the Commission confirmed the law deduced by Hélie in 1888 that f(V) is proportional to the sine of the ogival angle, at least for values from 40° to 90°, within the experimental errors.

In 1917, Chief Engineer Garnier of the Gâvre Commission published a table[18] of log B and the corresponding logarithmic derivative, with velocity as argument: this B is equivalent to f(V) for an ogival projectile with a 2-caliber radius head. This ideal shape is what we call Projectile Type 1 (see Figure 1). The Ordnance Department of the United States Army then prepared a table of the logarithm of the drag function[19]

$$G = kBV, \qquad (24)$$

with V expressed in meters per second, and the factor k to take account of differences in definitions and standards. Below 600 m/s, k = 0.00114; but at higher velocities, it is variable. G was called the Gâvre function; later, the symbol was changed to G_1 to denote that it pertains to Projectile Type 1.

In the table of log G, the argument, $V^2/100$, varies from to 0 to 33,000 (equivalent to 1817 m/s or 5961 fps). G has been tabulated for $V^2/100$ up to 25,000 (1581 m/s or 5187 fps).

* The tabulated trajectory data based on the old standard atmosphere give results that are nearly the same as those based on the new atmosphere providing a suitable change is made in the ballistic coefficient. For example, a 280-mm shell, fired with a muzzle velocity of 1795 fps, at an elevation of 45°, attains a range of 15,790 yds.; the ballistic coefficient is 3.62 with ICAO atmosphere and 3.50 with the old Ordnance Corps atmosphere. This function was used in computing trajectories before 1956, when the Department of Defense adopted the standard atmosphere of the International Civil Aviation Organization.

f. Ballistic Coefficient

The ballistic coefficient relative to the tabulated drag coefficient is defined by the formula

$$C_t = m/i_t d^2, \tag{22}$$

where m is the mass of the projectile in pounds,
d is the caliber (or maximum diameter) in inches.

g. Air Density

The air density is a function of altitude. The exponential function adopted by the Ordnance Corps* is tabulated in References 7 and 11. The ratio of the density at an altitude y to that at the surface is H(y).

4. DESCRIPTION OF TYPICAL PROJECTILES

This pamphlet contains data that pertain specifically to four types of projectiles: Types 1, 2 and 8 and the 90-mm HEAT Shell T108. They can be applied to other shapes by using a form factor, as explained in paragraph 3e.

a. Projectile Type 1

Most of the service projectiles used by the European artillery during the last third of the nineteenth century had square bases and ogival heads of various apical angles. In Russia, during the years 1868 and 1869, General Magenski conducted some experiments at St. Petersburg to determine the law of air resistance of projectiles with a semi-apical angle of 48°22′ (1.49-caliber radius head) at velocities from 172 to 409 m/s. In England, from 1866 to 1880, Bashforth conducted similar experiments with projectiles of the same form (it may be that the tables were reduced to this form) at velocities from 148 to 825 m/s. In Holland, Colonel Hojel conducted a few such experiments during 1883; the exact form is uncertain, but the ogive was evidently somewhat longer than those of the Russian experiments. In Germany, the Krupp factory conducted a large number of air resistance experiments at Meppen Proving Ground, starting in 1881 (the principal results were reported by Siacci in the "Rivista" for March 1896); the velocities varied from 368 to 910 m/s, and three different types of projectiles were used: one was the same as the Russian (or the results reduced to the same), but the others probably had longer ogives. In France, a commission of the naval artillery conducted many such experiments at Gâvre Proving Ground from 1873 to 1898 at velocities from 118 to 1163 m/s; they fired ogival projectiles with three semi-apical angles: 45°55′ (1.64 CRH), 41°40′ (1.98 CRH), and 31°45′ (3.34 CRH). For further details, see References 12 to 16.

In analyzing the results of the experiments,[17] the Gâvre Commission of Experiments assumed that the drag could be expressed in the form

$$D = \frac{\Delta}{g} a^2 V^2 f(V), \tag{23}$$

where

D—is the drag (kg),
Δ—the density of the air (kg/m^3),
g—the gravitational acceleration (m/sec^2),
a—the diameter of the largest cross section of the projectile (m),
V—the velocity (m/s).

The drag was treated as a function of velocity without taking account of the velocity of sound, although the dependence of drag on the velocity of sound was strongly suspected: the coefficient f was plotted as a function of V alone. After studying the results obtained with projectiles of various ogival angles, the Commission confirmed the law deduced by Hélie in 1888 that f(V) is proportional to the sine of the ogival angle, at least for values from 40° to 90°, within the experimental errors.

In 1917, Chief Engineer Garnier of the Gâvre Commission published a table[18] of log B and the corresponding logarithmic derivative, with velocity as argument: this B is equivalent to f(V) for an ogival projectile with a 2-caliber radius head. This ideal shape is what we call Projectile Type 1 (see Figure 1). The Ordnance Department of the United States Army then prepared a table of the logarithm of the drag function[19]

$$G = kBV, \tag{24}$$

with V expressed in meters per second, and the factor k to take account of differences in definitions and standards. Below 600 m/s, k = 0.00114; but at higher velocities, it is variable. G was called the Gâvre function; later, the symbol was changed to G_1 to denote that it pertains to Projectile Type 1.

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The product Gv has also been tabulated as a function of v in feet per second up to 6000 fps.[20]

b. Projectile Type 2

A sketch of Projectile Type 2 is shown in Figure 1. This projectile has a 6° boattail ½ caliber long and an ogivo-conical head 2.7 calibers long. The experimental 4.7-inch Shell E1 had this shape. However, the resistance firings were actually conducted with two types of 3.3-inch shell: Type 155 with a 5° boattail and Type 157 with a 7° boattail. The results were so close that they were grouped together to represent one type.

These projectiles were fired from 1922 to 1925 at velocities from 655 to 3187 fps. The results of these firings were fitted by the tabulated drag function G_2 (this is defined in paragraph 3d). Later, it was extrapolated to 10,000 fps by means of theoretical considerations. For the sake of greater smoothness and accuracy, two revisions have been made: $G_{2.1}$, tabulated to 6000 fps, and $G_{2.2}$, to 7000 fps. $G_{2.1}$ is approximately the same as G_2, but $G_{2.2}$ is lower at velocities above 2000 fps.

c. Projectile Type 8

A sketch of Projectile Type 8 is shown in Figure 1. This projectile has a square base and a secant ogive 2.18 calibers high. Its ogival radius is 10 calibers, which is twice that of a tangent ogive of the same height.

The drag function G_8 was calculated from data obtained by the British Ordnance Board from three sources:

(1) Measurements of a model in the National Physical Laboratory wind tunnel at Mach numbers from 0.22 to 0.89.

(2) Resistance firings of 2-pounder shell at Mach numbers from 0.65 to 2.8.

(3) Resistance firings of 3-inch shell at Mach numbers from 0.35 to 4.15.

G_8 was first tabulated to 5000 fps. Later, a revised table, called $G_{8.1}$, was extended to 6000 fps. The corresponding drag coefficients are approximately proportional to the British "1940 Law," which was first tabulated to 5000 fps and later revised above 3750 fps and extended to 6000 fps by a power law.

d. HEAT Shell, 90-mm, T108

A sketch of the 90-mm High Explosive Antitank Shell T108 is shown in Figure 1. This projectile has fins one caliber in span at the end of a long boom and a conical ogive about 2 calibers high.

The drag of this projectile was determined at velocities from 700 to 2700 fps by free flight firings in the large Spark Range of the Exterior Ballistic Laboratory at Aberdeen Proving Ground. The drag coefficient (defined in paragraph 3c) was fitted with a series of analytical expressions, which were used in tabulating it as a function of Mach number from 0.1 to 2.7.

5. Trajectories in Air

a. Equations of Motion

Under standard conditions, the equations of motion are

$$\ddot{x} = -\frac{GH}{C}\dot{x}, \quad \ddot{y} = -\frac{GH}{C}\dot{y} - g, \qquad (25)$$

where x is the horizontal range,

y—the altitude,

H—the air density ratio,

g—the gravitational acceleration (32.152 ft/sec^2),

and dots denote derivatives with respect to time. The initial conditions are

$$x_0 = 0,\ y_0 = 0,\ \dot{x}_0 = v_0 \cos \vartheta_0,\ \dot{y}_0 = v_0 \sin \vartheta_0,$$

where v_0 is the initial velocity,

ϑ_0—the angle of departure.

These equations cannot be solved analytically, but they can be solved by a method of numerical integration explained in References 21, 22, 23 and 24.

b. Characteristics of Trajectories

(1) *Ballistic Tables.* Trajectory data have been tabulated for Projectile Types 1 and 2 and for some bomb shapes.

The "Exterior Ballistic Tables Based on Numerical Integration" (References 7, 25, and 26) pertain to Projectile Type 1. The meter is the unit of length. The trajectories were computed forward and backward from the summit and cut off at several altitudes. If C_s is the "summital ballistic coefficient" of a trajectory, and y_s the maximum ordinate of a partial trajectory, the ballistic coefficient C of this partial trajectory satisfies the relation

$$\log_{10}C = \log_{10}C_s - 0.000{,}045\, y_s. \qquad (26)$$

For each y_s, C_s, and summital velocity v_s, Volume I gives the horizontal range, time, and velocity components at the beginning and end of the partial trajectory. If the muzzle velocity, angle of departure, and ballistic coefficient are known, Volume II may be used to find the time of flight to the summit, t_s, the velocity at the summit, the horizontal range to the summit, x_s, and the maximum ordinate. Then C_s can be computed and trajectory data can be found in Volume I for the given C_s and v_s at various altitudes.

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For each y_s, C_s, and summital velocity v_s, Volume I gives the horizontal range, time, and velocity components at the beginning and end of the partial trajectory. If the muzzle velocity, angle of departure, and ballistic coefficient are known, Volume II may be used to find the time of flight to the summit, t_s, the velocity at the summit, the horizontal range to the summit, x_s, and the maximum ordinate. Then C_s can be computed and trajectory data can be found in Volume I for the given C_s and v_s at various altitudes.

However, terminal values can be found more conveniently from Volume III, which gives the horizontal range X, time of flight T, angle of fall ω, and terminal velocity v_ω; the arguments are the same as in Volume II: the muzzle velocity varies from 80 to 920 meters per second, the angle of departure from 0° to 80° (at intervals of 1° from 0° to 30°, and 2° from 30° to 80°), the ballistic coefficient from 1.000 to 17.78 (at intervals of 0.05 in log C).

The "Brief Exterior Ballistic Tables for Projectile Type 1" (Reference 27) contains values that were obtained by interpolation in Volume III and conversion to English units (yards and feet per second). The muzzle velocity varies from 200 to 3000 feet per second in 100 foot per second increments. The angle of departure varies from 15° to 75° in 15° increments. Obviously, interpolation for angle of departure in this table is difficult; nevertheless, the table is useful for many purposes. The table of range for an elevation of 45°, which is nearly the maximum range, is reproduced here (Table I).

The "Exterior Ballistic Table for Projectile Type 2" (Reference 28) consists of three parts: Table I, Summital and Terminal Values; Table II, Coordinates as Functions of Time of Flight; and a large graph of Maximum Range vs. Ballistic Coefficient. The tables have been extended to include data for every 15° from 0° to 90°. The muzzle velocity varies from 1200 to 5200 feet per second, and the ballistic coefficient from 0.50 to 10.00. The coordinates are given in yards, the time in seconds, and the velocity in feet per second. Table I gives the time to the summit, the horizontal range to the summit, the maximum ordinate, the time of flight, the total range, the angle of fall, and the terminal velocity. Table II gives the horizontal range and altitude at every 5 seconds of time. Unfortunately, the velocity

TABLE I
MAXIMUM RANGE FOR PROJECTILE TYPE 1
ELEVATION 45°

Range in yards. Muzzle Velocity (v_o) in fps

Log C_1 . . . C_1 . . . v_o	0.000 1.000	0.050 1.122	0.100 1.259	0.150 1.413	0.200 1.585	0.250 1.778	0.300 1.995	0.350 2.239	0.400 2.512	0.450 2.818	0.500 3.162	0.550 3.548	0.600 3.981	0.650 4.467
200	397	398	399	400	401	403	404	405	406	407	408	408	409	409
300	847	856	864	871	877	883	888	892	897	900	904	907	910	913
400	1419	1441	1462	1481	1499	1514	1528	1541	1553	1564	1573	1582	1590	1597
500	2077	2121	2162	2201	2237	2270	2300	2328	2354	2378	2399	2418	2436	2452
600	2786	2862	2934	3003	3067	3126	3181	3232	3279	3322	3361	3397	3428	3461
700	3521	3638	3749	3856	3958	4053	4140	4222	4301	4372	4438	4500	4556	4606
800	4247	4411	4570	4724	4870	5008	5141	5267	5384	5494	5595	5690	5777	5859
900	4920	5136	5349	5556	5755	5947	6133	6310	6476	6634	6783	6923	7054	7174
1000	5489	5757	6024	6286	6542	6792	7037	7272	7498	7714	7919	8115	8299	8472
1100	5925	6238	6553	6866	7176	7482	7784	8079	8366	8644	8913	9172	9420	9658
1200	6251	6603	6958	7314	7671	8027	8381	8733	9077	9417	9749	10073	10388	10691
1300	6521	6905	7293	7689	8087	8488	8891	9292	9693	10092	10489	10872	11253	11626
1400	6759	7172	7594	8025	8460	8905	9353	9804	10258	10711	11165	11616	12065	12508
1500	6979	7419	7872	8336	8810	9294	9787	10287	10792	11301	11814	12329	12845	13360
1600	7185	7651	8134	8630	9141	9664	10200	10747	11302	11869	12442	13021	13605	14194
1700	7381	7873	8384	8912	9458	10021	10599	11193	11798	12421	13056	13701	14356	15019
1800	7569	8088	8627	9187	9767	10368	10989	11630	12288	12967	13662	14374	15099	15839
1900	7752	8296	8863	9454	10068	10707	11371	12059	12771	13505	14262	15041	15839	16659
2000	7931	8498	9093	9715	10364	11042	11749	12483	13247	14037	14859	15706	16580	17480
2100	8106	8698	9320	9972	10655	11372	12121	12903	13719	14567	15454	16372	17323	18306
2200	8278	8894	9542	10225	10942	11697	12489	13319	14188	15096	16049	17038	18068	19136
2300	8447	9086	9760	10473	11225	12018	12853	13732	14655	15624	16644	17706	18820	19975
2400	8612	9274	9975	10718	11505	12336	13215	14143	15121	16152	17239	18375	19574	20821
2500	8774	9459	10188	10961	11782	12652	13575	14552	15585	16679	17834	19048	20330	21675
2600	8935	9642	10397	11201	12056	12965	13931	14958	16048	17205	18430	19725	21095	22539
2700	9092	9823	10603	11438	13237	13275	14285	15362	16510	17731	19027	20405	21866	23412
2800	9247	10001	10807	11672	12595	13582	14637	15764	16970	18257	19625	21091	22645	24294
2900	9399	10177	11010	11903	12861	13886	14987	16165	17429	18782	20225	21778	23428	25184
3000	9549	10351	11210	12132	13123	14187	15334	16563	17885	19303	20823	22464	24213	26083

TABLE I (Continued)
MAXIMUM RANGE FOR PROJECTILE TYPE 1
ELEVATION 45°

Range in yards. Muzzle Velocity (v_o) in fps

Log C_1 C_1 v_o	0.600 3.981	0.650 4.467	0.700 5.012	0.750 5.623	0.800 6.310	0.850 7.079	0.900 7.943	0.950 8.913	1.000 10.00	1.050 11.22	1.100 12.59	1.150 14.13	1.200 15.85	1.250 17.78
200	409	409	409	410	410	410	411	411	412	412	412	413	413	413
300	910	913	915	917	918	919	921	923	924	925	925	926	927	928
400	1590	1597	1604	1611	1614	1619	1624	1627	1630	1634	1637	1639	1641	1643
500	2436	2452	2467	2480	2491	2501	2511	2519	2526	2533	2540	2545	2550	2555
600	3428	3461	3489	3513	3535	3555	3575	3591	3605	3619	3631	3640	3649	3659
700	4556	4606	4651	4695	4733	4766	4800	4829	4855	4879	4899	4918	4934	4951
800	5777	5859	5934	6001	6064	6120	6172	6219	6260	6298	6333	6364	6394	6418
900	7054	7174	7284	7389	7484	7573	7652	7725	7791	7852	7907	7958	8003	8044
1000	8299	8472	8634	8786	8928	9060	9180	9292	9394	9489	9575	9653	9724	9787
1100	9420	9658	9882	10094	10295	10484	10660	10825	10979	11120	11252	11373	11485	11588
1200	10388	10691	10984	11266	11536	11794	12038	12268	12487	12691	12882	13062	13228	13382
1300	11253	11626	11990	12344	12688	13020	13339	13644	13936	14213	14476	14723	14958	15175
1400	12065	12508	12945	13375	13796	14207	14604	14991	15364	15722	16067	16394	16702	16997
1500	12845	13360	13872	14381	14881	15374	15857	16331	16790	17235	17667	18080	18474	18852
1600	13605	14194	14783	15370	15956	16536	17109	17674	18227	18766	19291	19798	20287	20756
1700	14356	15019	15686	16357	17030	17702	18371	19031	19682	20324	20949	21559		
1800	15099	15839	16588	17344	18108	18875	19643	20406	21162	21913				
1900	15839	16659	17491	18338	19198	20063	20936	21809	22678	23543				
2000	16580	17480	18399	19340	20299	21270	22253	23243	24222	25222				
2100	17323	18306	19315	20353	21415	22497	23598	24710	25829	26957				
2200	18068	19136	20240	21380	22551	23748	24972	26217						
2300	18820	19975	21177	22421	23707	25027	26385	27771						
2400	19574	20821	22124	23478	24883	26334	27832	29372						
2500	20330	21675	23082	24553	26083	27673	29321	31028						
2600	21095	22539	24056	25646	27307	29048	30861	32748						
2700	21866	23412	25045	26763	28564	30458	32446							
2800	22645	24294	26040	27890	29849	31910	34091							
2900	23428	25184	27049	29037	31183	33410	35797							
3000	24213	26083	28080	30217	32501	34944	37558							

TABLE II
MAXIMUM RANGE OF PROJECTILE TYPE 2

Range in Units of 10 Yards

Muzzle Velocity fps	Log C_2								
	−0.3	−0.2	−0.1	0.0	0.2	0.4	0.6	0.8	1.0
1200	....	....	....	709	861	1004	1129	1227	1308
1600	....	....	....	851	1088	1335	1584	1821	2030
2000	....	....	....	1001	1322	1690	2091	2499	2880
2400	....	....	....	1143	1561	2071	2659	3295	3983
2800	759	905	1078	1284	1806	2479	3299	4322	5359
3200	819	983	1182	1426	2062	2926	4074	5589	7006
3600	877	1061	1287	1571	2335	3438	5061	7201	9034
4000	934	1139	1394	1720	2629	4056	6329	9106	11356
4400	992	1218	1504	1874	2959	4834	7917	11335	14019
4800	1051	1299	1617	2037	3342	5845	9836	13888	17015
5200	1112	1384	1735	2220	3795	7196	12091	16752	20333

components are not tabulated. The graph has a curve of maximum range vs. ballistic coefficient for every 400 feet per second muzzle velocity; it also lists the angle of departure for maximum range. A table for the maximum range is included in this pamphlet (Table II).

Some trajectories have been computed for Projectile Type 8 at an angle of departure of 45°. The range, which is nearly the maximum range, is tabulated here (Table III).

TABLE III

MAXIMUM RANGE FOR PROJECTILE TYPE 8
ELEVATION 45°

Range in Units of 10 Yards

Muzzle Velocity fps	Log C_8						
	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
1200	521	665	815	961	1091	1201	1288
2000	680	923	1223	1581	1934	2403	2826
2800	822	1168	1650	2287	3124	4231	5251
3600	964	1432	2145	3162	4865	6936	8741
4400	1108	1720	2726	4470	7492	10884	13661
5200	1258	2040	3424	6604	11693	16582	20319

When bombs were dropped at low speeds, trajectory data based on the Gâvre drag function G_1 were accurate enough for calibrating the sights. In fact, the descending branches of the trajectories in Volume I of the "Exterior Ballistic Tables" could be used for this purpose. Actually, some "Bomb Ballistic Reduction Tables" were published, which were more convenient for this purpose. However, at transonic speeds, the drag functions of modern bombs differ considerably from G_1. Drag coefficients have been determined for several shapes, but three of them have been selected as bases of "Bomb Ballistic Tables" (Reference 10): drag coefficient A was determined from wind tunnel tests of a shape designed by the National Advisory Committee for Aeronautics, and is probably the lowest drag coefficient that any bomb will have; drag coefficient B was determined from range bombing of the 3000-pound Demolition Bomb M119, and is suitable for other members of the series including the 750-pound Bomb M118 and the 10,000-pound Bomb M121; drag coefficient C was determined from resistance firings of the 75-mm Slug Mark I, and is probably the highest coefficient that any bomb will have.

The arguments of the "Bomb Ballistic Tables" are release altitude (1000 to 80,000 feet above sea level), true air speed of release in level flight (250 to 2000 knots), and performance parameter (0 to 1.5). The performance parameter P is inversely proportional to the ballistic coefficient; it is defined by the formula

$$P = 100d^2/m, \qquad (27)$$

where d is the maximum diameter of the bomb in feet.

m—the mass of the bomb in pounds.

The tables give the horizontal range from release to impact (feet), the time from release to impact (seconds), the striking velocity (ft/sec), and the striking angle (degrees). The drag coefficients are tabulated as functions of the Mach number in the appendix; they are graphed in the introduction.

(2) *Graphs.* Figures 3, 4 and 5 are graphs of the total time of flight for an elevation of 45°. The time of flight is useful in estimating wind effects.

For antiaircraft firing and for firing to the ground at different altitudes, points along the trajectory are needed. Figures 6 and 7 show the trajectories computed with G_2 and G_8 at angles of departure of 45° and 60° (there is a different graph for each muzzle velocity, each elevation, and each Projectile Type). Trajectories based on G_1 are not included because no antiaircraft projectiles have a blunt head. The T108 HEAT Shell is usually used only in direct fire at short ranges. Trajectory data for other elevations can be estimated from those at 45° or 60° by assuming that the slant range is the same for a given time of flight.

Unlike the vacuum trajectories, in air, the horizontal range to the summit is more than half the total range, and the time to the summit is less than half the total time; however, the maximum ordinate is approximately $(g/8)T^2$. In air, the terminal velocity is less than the initial velocity and the angle of fall is greater than the angle of departure. At a given elevation, the total range increases less rapidly than the square of the muzzle velocity; at a given muzzle velocity, the

range is a maximum at an angle somewhat different from 45°: lower for small ballistic coefficients, higher for large ones. However, the range at 45° is nearly as much as the maximum.

6. Siacci Tables

a. *Spin-stabilized Projectiles*

(1) *Tabular Functions.* In 1880, the Italian Col. Siacci introduced the Space, Time, Altitude, and Inclination Functions, (Reference 29), which simplify the calculation of trajectory data under certain restrictions. In particular, their use will be explained for three conditions: ground fire at low superelevations, antiaircraft fire (where the superelevation is small), and aircraft fire. These and some auxiliary functions based on G_1, $G_{2.2}$ (the second revision of G_2) and $G_{8.1}$ (the first revision of G_8) have been tabulated for the Air Force (References 30, 31 and 32). Since these Siacci tables were arranged primarily for use in computing aircraft firing tables, they were called Aircraft Tables; however, they can be used for other purposes, as explained below, in lieu of other forms. In these Aircraft Tables the argument is the component u along the line of departure of the velocity relative to the air, which is assumed motionless. The following functions and some of their derivatives with respect to u are tabulated:

Space $$p = \int_u^U \frac{du}{G(u)}, \tag{28}$$

Time $$t = \int_u^U \frac{du}{uG(u)}, \tag{29}$$

Inclination* $$q_1 = \int_u^U \frac{g\,du}{u^2G(u)}, \tag{30}$$

Altitude* $$q = \int_u^U \frac{q_1 du}{G(u)}, \tag{31}$$

Velocity increment $$\Delta' u = \frac{K_{D\delta}}{2} G(u), \tag{32}$$

$(K_{D\delta} = 16.4 \text{ per rad}^2)$,

Damping coefficient $$c'' = G(u)/2u. \tag{33}$$

The upper limit of integration, U, is 6000 fps for Projectile Type 1 and 7000 fps for Types 2 and 8. $K_{D\delta}$ is the yaw-drag coefficient, defined by the formula

$$K_D = K_{D_0}(1 + K_D\ \delta^2), \tag{34}$$

where K_{D_0} is the drag coefficient for 0 yaw,

δ—is the yaw in radians.

(2) *Ground Fire at Low Elevations*

(a) *Formulas.* Although the Aircraft Tables are not accurate in computing complete trajectories at high elevations, they can be used to obtain trajectory data at superelevations below 15° (for the derivation and application of the following formulas, see Reference 33). Here, we shall take

ρ/ρ_0—as the ratio of air density to normal, assumed constant for each problem,

a/a_0—the ration of the velocity of sound to normal, assumed constant for each problem.

If F is the temperature in degrees Fahrenheit,

$$a/a_0 = \sqrt{(459 + F)/518}. \tag{35}$$

Then the horizontal range is

$$x = (C\rho_0/\rho)\cos\vartheta_0\,[p(ua_0/a) - p(u_0a_0/a)], \tag{36}$$

the time of flight is

$$t_f = (C\rho_0 a_0/\rho a)\,[t(ua_0/a) - t(u_0a_0/a)], \tag{37}$$

the altitude is

$$y = x\tan\vartheta_0 - (C\rho_0 a_0/\rho a)^2\,[q(ua_0/a) - q(u_0a_0/a)] + x(C\rho_0a_0/\rho a)\sec\vartheta_0\,[q_1(u_0a_0/a)], \tag{38}$$

and the slope of the trajectory is

$$\tan\vartheta = \tan\vartheta_0 - (C\rho_0 a_0^2/\rho a^2)\sec\vartheta_0\,[q_1(ua_0/a) - q_1(u_0a_0/a)]. \tag{39}$$

The angle of departure for $y = 0$, may be found by formula

$$\sin\vartheta_0 = \frac{C\rho_0 a_0}{\rho a}\left[\frac{q(ua_0/a) - q(u_0a_0/a)}{p(ua_0/a) - p(u_0a_0/a)} - q_1(u_0a_0/a)\right]. \tag{40}$$

If $\rho/\rho_0 = 1$ and $a/a_0 = 1$, let $p = p(u)$, $p_0 = p(u_0)$, etc.

* In most places, the altitude and inclination functions are defined as twice these integrals.

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Then the horizontal range is

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the time of flight is

$$t_f = (C\rho_0a_0/\rho a)\,[t(ua_0/a) - t(u_0a_0/a)], \tag{37}$$

the altitude is

$$y = x\tan\vartheta_0 - (C\rho_0a_0/\rho a)^2\,[q(ua_0/a) - q(u_0a_0/a)] + x(C\rho_0a_0/\rho a)\sec\vartheta_0\,[q_1(u_0a_0/a)], \tag{38}$$

and the slope of the trajectory is

$$\tan\vartheta = \tan\vartheta_0 - (C\rho_0a_0^2/\rho a^2)\sec\vartheta_0\,[q_1(ua_0/a) - q_1(u_0a_0/a)]. \tag{39}$$

The angle of departure for $y = 0$, may be found by formula

$$\sin\vartheta_0 = \frac{C\rho_0a_0}{\rho a}\left[\frac{q(ua_0/a) - q(u_0a_0/a)}{p(ua_0/a) - p(u_0a_0/a)} - q_1(u_0a_0/a)\right]. \tag{40}$$

If $\rho/\rho_0 = 1$ and $a/a_0 = 1$, let $p = p(u)$, $p_0 = p(u_0)$, etc.

* In most places, the altitude and inclination functions are defined as twice these integrals.

Then these formulas can be expressed more simply:

$$x = C \cos \vartheta_o (p - p_o), \quad (36a)$$

$$t_f = C(t - t_o), \quad (37a)$$

$$y = x \tan \vartheta_o - C^2(q - q_o) + C \sec \vartheta_o \, x \, q_{1o}) \quad (38a)$$

$$\tan \vartheta = \tan \vartheta_o - C \sec \vartheta_o (q_1 - q_{1o}), \quad (39a)$$

and, for $y = 0$,

$$\sin \vartheta_o = C \left[\frac{q - q_o}{p - p_o} - q_{1o}\right] \quad (40a)$$

(b) *Example*

Data: Tank Gun, 76-mm, M1A2; Shot, AP, M339.

Caliber	:	3.000 in.
Mass	:	14.58 lb.
Muzzle Velocity	:	3200 fps
Form Factor	:	1.164 on $G_{8.1}$
Air Density Ratio	:	1.000
Sound Velocity Ratio	:	1.000

Problem: To find the range, time of flight, angle of departure, and angle of fall where the velocity is 2000 fps.

Solution: From the Aircraft Table Based on $G_{8.1}$:

u	p	t	q	q_1
2000	23079.3	6.5132	489.0	.06523
3200	15425.1	3.4997	161.6	.02637
Diff.	7654.2	3.0135	327.4	.03886

$C_{8.1} = 14.58/1.164\ (3.000)^2 = 1.392$ (Eq. 22)

$\sin \vartheta_o = 1.392\ (327.4/7654.2 - .02637) = .02283$ (Eq. 40a)

$\vartheta_o = 23.26$ mils, $\cos \vartheta_o = .99974$, sec. $\vartheta_o = 1.00027$

$\tan \vartheta_o =$ .02284

$1.392(1.00027)(.03886) =$ .05411

$\tan \vartheta =$ —.03127 (Eq. 39a)

$\vartheta = -31.84$ mils

$x = 1.392\ (.99974)(7654.2) = 10652$ ft (Eq. 36a)

$t_f = 1.392\ (3.0135) = 4.19$ sec (Eq. 37a)

NOTE: Since u is the component of the velocity along the line of fire, the tangential velocity is $v = u \cos \vartheta_o / \cos \vartheta$.

In this example, $v = 2000\ (.99974)/.99951 = 2000.5$ fps.

(3) *Antiaircraft Fire*

(a) *Formulas.* These tables can also be used for the flat part of antiaircraft trajectories, where the superelevation is less than 15° (for the derivation and application of the following formulas, see Reference 33). Let ρ/ρ_o and possibly C be known variable functions of x, but assume a constant value of a/a_o (either 1 or an average). Since ρ/ρ_o is a function of y, denoted by H, its relation to x can be found by the approximate substitution

$$y \approx x \tan \vartheta_o. \quad (41)$$

Let $m = \tan \vartheta$, $m_o = \tan \vartheta_o$.

Then the following trajectory data can be found by Simpson's rule or some other method of numerical integration:

$$p\,(ua_o/a) = p(u_o a_o/a) + \sec \vartheta_o \int_0^x \frac{H}{C}\, dx, \quad (42)$$

$$t_f = \sec \vartheta_o \int_0^x \frac{dx}{u}, \quad (43)$$

$$m = m_o - \int_0^x \frac{g \sec^2 \vartheta_o}{u^2}\, dx, \quad (44)$$

$$y = \int_0^x m\, dx. \quad (45)$$

Check: $x \tan \vartheta_o = 10652\ (.02284) =$	243.29
$-C_{8.1}^2 (q - q_o) = (1.392)^2\ (327.4) =$	-634.40
$C_{8.1} \sec \vartheta_o \times q_{1o} = 1.392\ (1.00027)\ (10652)\ (.02637) =$	391.11
$y =$	0.00 (Eq. 38a)

(b) *Example*

Data: Automatic (AA) Gun, 40-mm, M2A1; Shot, AP, M81
Muzzle Velocity : 2870 fps
Ballistic Coefficient : 0.574 on $G_{8.1}$
Angle of Departure : 1200 mils
Air Density Ratio : $H = e^{-hy}$
Sound Velocity Ratio: 1.000
Problem: To find the time of flight and altitude for horizontal ranges from 0 to 2400 ft.
Solution: (See remarks below and use the Aircraft Table Based on $G_{8.1}$).

x ft 1	Mhy est 2	H 3	$\int H dx$ 4	p(u) 5	u fps 6	$\frac{2613.1}{u}$ 7	t_f sec 8	$\left(\frac{2613.1}{u}\right)^2$ 9	$\int\left(\frac{2613.1}{u}\right)^2 dx$ 10	m 11	y ft 12
0	.00000	1.0000	0.00	17439.6	2870.0	0.9105	0.000	0.8290	0	2.4142	0
300	.00993	.9774	....	....	......	......	.....	......	....	......	....
600	.01987	.9553	586.49	20109.5	2446.4	1.0681	0.594	1.1408	591	2.3952	1443
900	.02980	.9337	....	....	......	......	.....	......	....	......	....
1200	.03973	.9126	1146.76	22660.1	2060.6	1.2681	1.295	1.6081	1416	2.3687	2872
1500	.04966	.8919	....	....	......	......	.....	......	....	......	....
1800	.05960	.8718	1681.96	25096.6	1716.3	1.5225	2.132	2.3180	2593	2.3308	4282
2100	.06953	.8521	....	....	......	......	.....	......	....	......	....
2400	.07946	.8328	2193.26	27424.2	1413.7	1.8484	3.143	3.4166	4314	2.2755	5664

Remarks:

Col. 1. The interval of x should be chosen small enough so that the integrations may be performed accurately by Simpson's rule: not more than 600 feet.

Col. 2. $M = \log_{10} e = 0.43429$. $h = 0.000{,}031{,}58\ ft^{-1}$. The estimate of $y = x \tan \vartheta_0$. A slightly smaller factor would be better at the longer ranges.

Col. 3. For convenience, the formula $H = \text{anti-colog}_{10}\ Mhy$ was used. The density function H(y) has been tabulated (Reference 11); it gives the same values. The ICAO "Standard Atmosphere—Tables and Data"[1] gives slightly different values.

Col. 4. Simpson's rule was used, but the trapezoid rule gives 2193.30 for the last value.

Col. 5. The first value was found in the Aircraft Table for $u_0 = 2870$ fps. The other values were computed by Eq. (42) with

$$\sec \vartheta_0 / C_{8.1} = 2.6131/0.574 = 4.5524.$$

Col. 6. The values of u after the first were found in the Aircraft Table by inverse interpolation, using the first two terms of Taylor's series.

Col. 7. $\sec \vartheta_0 = 2.6131$. This is multiplied by 1000 to avoid writing zeros.

Col. 8. Eq. (43). Simpson's rule gives 3.131 for the last value, but the trapezoid rule was used in order to obtain intermediate values.

Col. 9. The square of Col. 7.

Col. 10. Note the factor 10^6 in these values. Simpson's rule gives 4259 for the last value, but the trapezoid rule was used so as to obtain more values.

Col. 11. Computed by Eq. (44) with

$$m_0 = \tan \vartheta_0,\ g = 32.152\ ft/sec^2.$$

Col. 12. Computed by Eq. (45). Simpson's rule gives 5666 ft for the final value.

(4) *Aircraft Fire*

(*a*) *Formulas.* These tables are especially useful for aircraft fire (for the derivation and application of the following formuas, see References 34 and 35). Before using them, the initial conditions must be determined. Let

A—be the azimuth, measured from the direction of motion of the airplane,

Z—the zenith angle, measured from the upward vertical,

v_0—the muzzle velocity (relative to the gun),

w—the true air speed,

u_0—the initial velocity relative to the air,

δ_0—the initial yaw relative to the motion of the air.

The initial velocity and yaw may be found by the formulas

$$u_o^2 = v_o^2 + w^2 + 2wv_o \sin Z \cos A, \tag{46}$$

$$\delta_o^2 = (1 - \sin^2 Z \cos^2 A)\, w^2/u_o^2\ (\text{rad})^2. \tag{47}$$

In *forward* firing, $A = 0$; $Z = 1600$ mils,

$$u_o = v_o + w, \tag{46a}$$

$$\delta_o = 0. \tag{47a}$$

In *rearward* firing, $A = 3200$ mils, $Z = 1600$ mils,

$$u_o = v_o - w, \tag{46b}$$

$$\delta_o = 0. \tag{47b}$$

The air density ratio H and the sound velocity ratio a/a_o may be taken as constants. Approximately, if Y is the altitude of the airplane above sea level,

$$H = e^{-hY},\ h = 0.000{,}031{,}58\ \text{ft}^{-1}, \tag{48}$$

$$a/a_o = e^{-kY},\ k = 0.000{,}003{,}01\ \text{ft}^{-1}. \tag{49}$$

If s_s is the standard stability factor, which corresponds to the muzzle velocity, muzzle spin, and standard surface air density, the initial stability factor is

$$s_o = s_s v_o^2/u_o^2\, H. \tag{50}$$

If h is the yawing moment damping factor, $\varkappa$—the cross wind force damping factor,

$$c' = (h + \varkappa)/2u, \tag{51}$$

the loss in velocity due to yaw is

$$\Delta u = \frac{\delta_o^2 a}{a_o}\ \frac{(s_o - 1/2)\, \Delta' u\, (u_o/a)}{(s_o - 1)\, Cc' + c''\, (u_o a_o/a)} \tag{52}$$

Then the effective initial velocity is

$$u_e = u_o - \Delta u. \tag{53}$$

Now we can find the distance measured along the line of departure to a point directly above the projectile,

$$P = (C/H)\, [p(ua_o/a) - p\,(u_e a_o/a)], \tag{54}$$

the drop of the projectile,

$$Q = (Ca_o/Ha)^2\, [q(ua_o/a) - q(u_e a_o/a)] - (Ca_o/Ha)\, Pq_1\, (u_e a_o/a), \tag{55}$$

and the time of flight is,

$$t_f = (Ca_o/Ha)\, [t(ua_o/a) - t\,(u_e a_o/a)]. \tag{56}$$

The horizontal and vertical components, if desired, may be found by the formulas

$$x = P \cos \vartheta_o, \tag{57}$$

$$y = P \sin \vartheta_o - Q, \tag{58}$$

with the angle of departure determined by the formula

$$\tan^2 \vartheta_o = \frac{\cos^2 Z}{(\sin Z \sin A)^2 + (\sin Z \cos A + w/v_o)^2} \tag{59}$$

or

$$\cot^2 \vartheta_o = (\tan Z \sin A)^2 + (\tan Z \cos A + w/v_o \cos Z)^2. \tag{59a}$$

Illustrative example is given on the following page.

(b) *Example*

Data:

Azimuth	:	1200 mils
Zenith Angle	:	800 mils
Muzzle Velocity	:	3000 fps
Air Speed	:	1000 fps
Altitude	:	30,000 ft
Standard Stability Factor	:	3.66
Damping Factors h + x	:	10.6 sec^{-1} at 3000 fps
Ballistic Coefficient	:	0.235 on $G_{8.1}$

Problem: To find the time of flight and drop for a Siacci range (P) at 1200 ft.

Solution: (Use the Aircraft Table Based on $G_{8.1}$).

$\cos A = 0.38268$

$\sin Z = 0.70711$

$\sin Z \cos A = 0.27060$

$2wv_o = 2 \times 1000 \times 3000 = 6{,}000{,}000$

$v_o^2 = (3000)^2 = 9{,}000{,}000$

$w^2 = (1000)^2 = 1{,}000{,}000$

$2wv_o \sin Z \cos A = 1{,}623{,}600$

$u_o^2 = 11{,}623{,}600$ (Eq. 46)

$u_o = 3409.3$ fps

$\delta_o^2 = 0.72940 \times 1{,}000{,}000/11{,}623{,}600 = 0.06275$ (Eq. 47)

$\delta_o = 0.2505$ rad (255.2 mils), (1 rad = 1018.6 mils).

$hY = 0.000{,}031{,}58 \times 30{,}000 = 0.9474$

$H = 0.3877$ (Eq. 48)

$kY = 0.000{,}003{,}01 \times 30{,}000 = 0.0903$

$a/a_o = 0.9137$ (Eq. 49)

$s_o = 3.66 \times 9{,}000{,}000/11{,}623{,}600 \times 0.3877 = 7.31$ (Eq. 50)

$c' = 10.6/2 \times 3000 = 0.00177\ ft^{-1}$

$u_o a_o/a = 3409.3/0.9137 = 3731.3$ fps

$\Delta' u(u_o a_o/a) = 1.4003\ sec^{-1}$ (from Aircraft Table)

$c''(u_o a_o/a) = 2288 \times 10^{-8}\ ft^{-1}$ (from Aircraft Table)

$$\Delta u = 0.06275 \times 0.9137 \frac{6.81 \times 1.4003}{6.31 \times 0.235 \times 0.00177 + 0.000{,}022{,}88}$$

$$= 0.05733 \frac{9.536}{0.002{,}625 + 0.000{,}023} = 206.5 \text{ fps (Eq. 52)}$$

$u_e = 3409.3 - 206.5 = 3202.8$ fps (Eq. 53)

$C_{8.1}/H = 0.235/0.3877 = 0.6061$

$C_{8.1}a_o/Ha = 0.6061/0.9137 = 0.6633$

$(C_{8.1}a_o/Ha)^2 = (0.6633)^2 = 0.4400$

$PH/C_{8.1} = 1200/0.6061 = 1979.9$ ft

$u_e a_o/a = 3202.8/0.9137 = 3505.3$ fps

Find $p(u_e a_o/a)$ in Table, then $p(ua_o/a)$ by Eq. 54.

ua_o/a	p	t	q	q_o
3174.3	15580.5	3.548	165.7	.02686
3505.3	13600.6	2.955	118.4	.02114
	1979.9	0.593	47.3	.00572

$t_f = 0.6633 \times 0.593 = 0.393$ sec (Eq. 56)

$0.4400 \times 47.3 = 20.8$

$0.6633 \times 1200 \times 0.02114 = 16.8$

$Q = 4.0$ ft (Eq. 55)

(c) *Interpolation.* The derivatives listed in the Aircraft Tables should be used for interpolating backward or forward from the nearest tabular value of the argument. For instance, in the last example, it is desired to find p for u = 3505.3 and then to find u for p = 15580.5. First, the table gives

at $u = 3510,\quad p = 13572.81,\quad \frac{dp}{du} = -5.923$

Hence, $\Delta u = -4.7,\quad \Delta p = +27.84,$

and, at $u = 3505.3,\quad p = 13600.65.$

Then we have $p = 15580.5$

and the table gives

$p = 15606.2,\ \frac{dp}{du} = -6.044$ at $u = 3170.$

Hence, $\Delta p = -25.7 \quad \Delta u = +4.3,$

and $u = 3174.3.$

b. Fin-stabilized Projectiles

(1) *Tabular Functions.* Reference 6 contains a table of modified Siacci functions based on the drag coefficient for the 90-mm High Explosive Antitank Shell T108, which has rigid fins. These tables can be used for other similar projectiles, although they are not suitable for all fin-stabilized projectiles. The following functions and their derivatives are tabulated as functions of Mach number M:

$$\text{Space} \qquad S = \int_{M}^{2.7} \frac{dM}{MK_D(M)}, \tag{60}$$

$$\text{Time} \qquad T = \int_{M}^{2.7} \frac{dM}{M^2K_D(M)}, \tag{61}$$

$$\text{Inclination} \qquad I = \frac{g}{a_o}\int_{M}^{2.7} \frac{dM}{M^3K_D(M)}, \tag{62}$$

$$\text{Altitude} \qquad A = \int_{M}^{2.7} \frac{I(M)dM}{MK_D(M)}. \tag{63}$$

The factor in the inclination function is

$g/a_o = 32.154/1120.27 = 0.0287 \text{ sec}^{-1}$.

(2) *Application*

(a) *Formulas.* The following formulas are applicable for computing trajectory data at superelevations below 15°.

Let

m be the mass of the projectile in pounds,

i—the form factor of the projectile,

d—the maximum diameter of the projectile in feet,

v—the velocity of the projectile in feet per second,

v_o—the muzzle velocity in feet per second,

a—the velocity of sound in air in feet per second,

ρ—the density of air in pounds per cubic foot,

ϑ—the angle of inclination of the trajectory,

ϑ_o—the angle of departure.

Also let

$a_o = 1120.27$ fps,

$\rho_o = 0.07513 \text{ lb/ft}^3$,

$$M_o = v_o/a, \tag{64}$$

$$\overline{M} = v \cos\vartheta / a \cos\vartheta_o, \tag{65}$$

$$\gamma = \rho_o i d^2/m. \tag{66}$$

Then the horizontal range in feet is

$$x = (\rho_o/\rho\gamma)\,[S(\overline{M}) - S(M_o)], \tag{67}$$

the time of flight in seconds is

$$t_f = (\rho_o \sec\vartheta_o/a\rho\gamma)\,[T(\overline{M}) - T(M_o)], \tag{68}$$

the altitude in feet is

$$y = x\tan\vartheta_o - \frac{a_o\rho_o^2\sec^2\vartheta_o}{a^2\rho^2\gamma^2}[A(\overline{M}) - A(M_o)] + x\frac{a_o\rho_o\sec^2\vartheta_o}{a^2\rho\gamma}I(M_o). \tag{69}$$

$$\tan\vartheta = \tan\vartheta_o - \frac{a_o\rho_o\sec^2\vartheta_o}{a^2\rho\gamma}[I(\overline{M}) - I(M_o)]. \tag{70}$$

The angle of departure for $y = 0$ may be found by the formula

$$\sin 2\vartheta_o = \frac{a_o\rho_o}{a^2\rho\gamma}\left[\frac{A(\overline{M}) - A(M_o)}{S(\overline{M}) - S(M_o)} - I(M_o)\right]. \tag{71}$$

If $\rho = \rho_o$ and $a = a_o$, let $S = S(\overline{M})$, $S_o = S(M_o)$, etc. Then these formulas can be expressed more simply:

$$x = (1/\gamma)(S - S_o), \tag{67a}$$

$$t_f = (\sec.\vartheta_o/a_o\gamma)(T - T_o), \tag{68a}$$

$$y = x\tan\vartheta_o - \frac{\sec^2\vartheta_o}{a_o\gamma^2}(A - A_o) + x\frac{\sec^2\vartheta_o}{a_o\gamma}I_o \tag{69a}$$

$$\tan\vartheta = \tan\vartheta_o - \frac{\sec^2\vartheta_o}{a_o\gamma}(I - I_o), \tag{70a}$$

and for $y = 0$,

$$\sin 2\vartheta_o = \frac{2}{a_o\gamma}\left[\frac{A - A_o}{S - S_o} I_o\right]. \tag{71a}$$

(b) *Example*

Data: Gun, 90-mm; Shell, HEAT, T108E20

Caliber	: 0.29525 ft
Mass	: 14.2 lb
Muzzle Velocity	: 2400 fps
Form Factor	: 1.00 on G_{T108}
Air Density	: 0.07513 lb/ft^3
Sound Velocity	: 1120.27 fps

Problem: To find the velocity, time of flight, angle of departure, and angle of fall at a range of 3000 feet.

Solution: From the Table of the Siacci Functions based on Shell, 90-mm, HEAT, T108.

M	S	T	A	I
1.73650	3.09196	1.427502	.02537	.01923
2.14234*	1.70836	0.709983	.00670	.00851
Dif.	1.38360**	0.717519	.01867	.01072

$* \ M_o = 2400/1120.27 = 2.14234$ (Eq. 64)

$\gamma = 0.07513(1.00)(0.29525)^2/14.2 = 0.000{,}461{,}2 \text{ ft}^{-1}$ (Eq. 66)

$a_o\gamma = 1120.27(0.000{,}461{,}2) = 0.51667 \text{ sec}^{-1}$

$a_o\gamma^2 = 0.51667(0.000{,}461{,}2) = 0.000{,}238.29 \text{ ft}^{-1}\text{ sec}^{-1}$

$**S - S_o = \gamma x = 0.000{,}461{,}2(3000) = 1.38360$ (Eq. 67a)

$$\sin 2\vartheta_o = \frac{2}{0.51667}\left[\frac{0.01867}{1.38360} - 0.00851\right] = 0.01928 \text{ (Eq. 71a)}$$

$2\vartheta_o = 19.64$ mils, $\vartheta_o = 9.82$ mils, $\cos\vartheta_o = 0.99995$,

$\sec\vartheta_o = 1.00005$, $\sec^2\vartheta_o = 1.00010$

$$\begin{aligned} \tan\vartheta_o &= \quad 0.00964 \\ -\frac{1.00010}{0.51667}(0.01072) &= -\underline{0.02075} \\ \tan\vartheta &= -0.01111 \text{ (Eq. 70a)} \end{aligned}$$

$\vartheta = -11.32$ mils, $\cos\vartheta = 0.9994$

$v = 1120.27\ (1.73650)\ (0.99995)/(0.99994) = 1945.37$ fps (Eq. 65)

$$t_f = \frac{1.00005}{0.51667}(0.717519) = 1.389 \text{ sec (Eq. 68a)}$$

Check: $x \tan\vartheta_o = 3000\ (.00964) = 28.92$ ft

$$-\frac{\sec^2\vartheta_o(A - A_o)}{a_o\gamma^2} = -\frac{1.00010(0.01867)}{0.000{,}238.29} = -78.36 \text{ ft}$$

$$\frac{x\sec^2\vartheta_o}{a_o\gamma} I_o = \frac{3000(1.00010)}{0.51667}(0.00851) = +49.42 \text{ ft}$$

$y = 28.92 + 49.42 - 78.36 = -0.02$ ft (Eq. 69a).

7. Differential Effects

Some of the effects on range and deflection caused by variations from standard conditions will be discussed and explained. More detailed explanations are given in References 21, 22, 23 and 24.

a. Effects on Range

(1) *Height of Target.* Two methods may be used for determining the effect of the height of the target with respect to the gun: one by means of the trajectory chart, and another by a correction to the angle of departure.

(a) For large variations in height, the trajectory chart should be used. The trajectory that passes through the point determined by the given horizontal range and altitude can be found. The total range of this trajectory is the range where it crosses the axis. The difference between the given range and the total range is the effect of the height of target. For greater accuracy, the exterior ballistic tables may be used in lieu of the charts.

(b) For small variations in height, an approximate effect may be found by adding the angle of site,

$$\epsilon = \tan^{-1}(H/R_o), \tag{72}$$

to the angle of departure φ_o, obtaining the corrected angle

$$\varphi = \varphi_o + \epsilon. \tag{73}$$

Here, H is the height of target,

R_o—the map range,

φ_o—the angle of departure corresponding to the map range in the Exterior Ballistic Tables,

φ—the angle of departure corresponding to the corrected range R.

Then the effect on range due to height of target is

$$\Delta_H R = R_o - R. \tag{74}$$

(b) *Example*

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$$\tan\vartheta_o = \quad 0.00964$$

$$-\frac{1.00010}{0.51667}(0.01072) = \quad -0.02075$$

$$\tan\vartheta = \quad -0.01111 \text{ (Eq. 70a)}$$

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Some of the effects on range and deflection caused by variations from standard conditions will be discussed and explained. More detailed explanations are given in References 21, 22, 23 and 24.

a. Effects on Range

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(a) For large variations in height, the trajectory chart should be used. The trajectory that passes through the point determined by the given horizontal range and altitude can be found. The total range of this trajectory is the range where it crosses the axis. The difference between the given range and the total range is the effect of the height of target. For greater accuracy, the exterior ballistic tables may be used in lieu of the charts.

(b) For small variations in height, an approximate effect may be found by adding the angle of site,

$$\epsilon = \tan^{-1}(H/R_o), \qquad (72)$$

to the angle of departure φ_o, obtaining the corrected angle

$$\varphi = \varphi_o + \epsilon. \qquad (73)$$

Here, H is the height of target,

R_o—the map range,

φ_o—the angle of departure corresponding to the map range in the Exterior Ballistic Tables,

φ—the angle of departure corresponding to the corrected range R.

Then the effect on range due to height of target is

$$\Delta_H R = R_o - R. \qquad (74)$$

(2) *Elevation.* Since the Exterior Ballistic Tables give the range as a function of elevation for constant values of muzzle velocity and ballistic coefficient, the effect of an increase in elevation may be found from the differences. At short ranges, the effect may be computed more accurately by using the Siacci tables, as explained above. The vacuum formula (Eq. 6) may be useful in estimating the effect, especially for large ballistic coefficients. It should be noted that the effect is small in the vicinity of the maximum range and, of course, is negative at elevations above that corresponding to the maximum range.

(3) *Muzzle Velocity.* Since the Exterior Ballistic Tables give the range as a function of muzzle velocity for constant values of elevation and ballistic coefficient, the effect of an increase in muzzle velocity may be found from the differences. At short ranges, the effect may be computed by using the Siacci tables. In vacuum, the range is proportional to the square of the velocity, as shown in Equation 6; in air, this is a good approximation if the ballistic coefficient is large.

(4) *Ballistic Coefficient.* Since the Exterior Ballistic Tables give the range as a function of ballistic coefficient for constant values of muzzle velocity and elevation, the effect of an increase in log C may be found from the differences. At short ranges, the effect may be computed by using the Siacci tables. It is customary to find the effect of a 1% increase in C, which is equivalent to an increase of 0.00432 in log C.

(5) *Weight of Projectile.* An increase in weight of projectile decreases the muzzle velocity and increases the ballistic coefficient.

The effect on muzzle velocity may be expressed

$$\Delta_p v_o = n v_o (\Delta p)/p, \tag{75}$$

where v_o is the muzzle velocity

p is the projectile weight,

n the logarithmic differential coefficient.

The value of n may be obtained from interior ballistic tables. It is approximately -0.3 for a rifled gun with multiperforated propellant grains, -0.4 for a rifled gun with single-perforated grains, -0.47 for a smooth bore mortar with flake propellant, and -0.65 for a recoilless rifle with multiperforated grains.

Since the ballistic coefficient is proportional to the projectile weight by definition, Equation (22), the effect on C may be expressed

$$\Delta_p C = C(\Delta p)/p. \tag{76}$$

If $\Delta_v R$ is the effect on range of 1 unit increase in muzzle velocity, $\Delta_C R$ the effect on range of 1% increase in ballistic coefficient, the effect on range of an increase in projectile weight is

$$\Delta_p R = \Delta_v R(\Delta_p v) + \Delta_C R(100\ \Delta_p C)/C. \tag{77}$$

(6) *Air Density.* The retardation due to air resistance is proportional to the air density and inversely proportional to the ballistic coefficient, as indicated in Eq. (25). Therefore, to a first approximation, the effect on range of an increase in air density is equal to the effect of the same percentage decrease in C.

(7) *Air Temperature.* An increase in the temperature of the atmosphere affects the range in two ways: it increases the density of the air and the velocity of sound. The temperature of the air may affect the temperature of the propellant, which affects the muzzle velocity, but that is an interior ballistic problem.

For a given pressure and humidity, the density is proportional to the absolute temperature. The effect of density on range has been treated above.

For a given humidity, the velocity of sound is proportional to the square root of the absolute temperature. For a given projectile velocity, the Mach number is inversely proportional to the acoustic velocity. A change in M produces a change in the drag function. A new trajectory could be computed with the varied function; however, it is simpler to use an approximate formula deduced from a semi-empirical formula derived by Dr. Gronwall:[36]

$$\Delta_\tau R = 0.1929\ [0.01R - 0.005 v_o \Delta_v R + (1 - 0.000{,}015 y_s)\Delta_C R]\Delta\tau, \tag{78}$$

where R is the range,

$\Delta_\tau R$ —the increase in range due to increase of temperature

v_o—the muzzle velocity in feet per second,

$\Delta_v R$—the effect on R of 1 fps increase in v_o,

y_s—the maximum ordinate in feet,

$\Delta_C R$—the effect on R of 1% increase in C,

$\Delta\tau$—the increase in temperature in degrees Fahrenheit.

(8) *Wind.* A rear wind has three effects: it moves the coordinate system along with it relative to the earth; it increases the angle of departure relative to the air; and it decreases the muzzle velocity relative to the air. The total effect on range may be expressed as follows:

$$\Delta_w R = W_R \left[T + \frac{\sin \varphi}{v_0} \frac{\Delta \varphi R}{\Delta \varphi} - \cos \varphi \frac{\Delta_v R}{\Delta v_0} \right], \tag{79}$$

where W_R is the range component of the wind (a rear wind is positive; a head wind is negative),

T—the time of flight,

φ—the angle of departure,

v_0—the muzzle velocity,

$\Delta\varphi R$—the effect on range due to a change of $\Delta\varphi$ radians in φ,

$\Delta_v R$—the effect on range due to a change Δv_0 in v_0.

b. Effects on Deflection

(1) *Wind.* A crosswind has two effects: it moves the coordinate system with it and it changes the direction of motion relative to the air. The total effect on linear deflection may be expressed

$$\Delta_w Z = W_z (T - R/v_0 \cos \varphi), \tag{80}$$

where W_z is the crosswind.

The angular deflection due to wind, in radians, is

$$D_w = W_z (T/R - 1/v_0 \cos \varphi). \tag{81}$$

(2) *Drift.* After the yaw due to initial disturbances has damped out, a spinning shell has a yaw of repose due to the curvature of the trajectory. This yaw is mainly to the right of the trajectory if the shell has right-handed spin; to the left if it has left-handed spin. It may be determined at short ranges by firing alternately from barrels with right and left hand twists of rifling, or at long ranges by correcting the observed deflection for the effects of wind, cant of trunnions, and rotation of earth.

The differential equation of motion for the drift z is[28]

$$\ddot{z} = \frac{2\pi g A v_0 L N}{m d n u^2 M N_0} \dot{x} - \frac{G \rho \dot{z}}{\rho_0 C}, \tag{82}$$

where g is the gravitational acceleration,

A—the axial moment of inertia,

v_0—the muzzle velocity,

L—the crosswind force (lift),

N—the spin,

N_0—the initial spin,

m—the mass of the projectile,

d—the caliber,

u—the velocity of the projectile relative to the air,

M—the overturning moment about the center of gravity,

x—the horizontal range,

G—the drag function,

ρ—the air density,

ρ_0—the standard air density (0.07513 lb/ft^3),

C—the ballistic coefficient.

Dots denote time derivatives, and the initial conditions are

$\dot{z}_0 = 0, z_0 = 0.$

c. Ballistic Density, Temperature, and Wind

In treating the effects of variations in the atmosphere, it has been assumed that the percentage variations from standard density and temperature and the wind are constant all along the trajectory. Actually, they vary with altitude, so that weighted mean values must be used. The weighting factors are determined by the proportion of the total effect that a variation occurring in each altitude zone would have. Instead of computing a set of weighting factors for each trajectory, a few representative curves are used. The sum of the products of the zonal variations by their weighting factors is called the ballistic density, temperature, or wind. The results are then treated as constants in computing the effects on range and deflection.

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GLOSSARY

angle of departure. (elevation) As used in this pamphlet these terms mean *quadrant angle of departure* which is defined as the angle between the horizontal and the line along which the projectile leaves the projector. One of the two initial conditions of a trajectory, the other condition being *initial velocity.*

For a rocket-fired projectile in which the propellant burning continues after the rocket leaves the launcher, it is necessary to make assumptions relative to the point at which the rocket becomes a missile in free flight and the velocity attained at that point.

angle of inclination. The angle between the horizontal and the tangent to the trajectory at any point. At the origin of the trajectory this is the same as the *angle of departure.*

apical (semi-apical) angle. In general the angle formed at the apex or tip of anything. As applied to projectiles, the angle between the tangents to the curve outlining the contour of the projectile at its tip, or for semi-apical angle, the angle between the axis and one of the tangents. For a projectile having a conical tip, the cone apex angle.

ballistics. Branch of applied mechanics which deals with the motion and behavior characteristics of missiles, that is, projectiles, bombs, rockets, guided missiles, etc., and of accompanying phenomena. It can be conveniently divided into three branches:—*interior ballistics,* which deals with the motion of the projectile in the bore of the weapon; *exterior ballistics,* which deals with the motion of the projectile while in flight; and *terminal ballistics,* which is concerned with the effect and action of the projectile when it impacts or bursts.

crosswind force. The component of the force of air resistance acting in a direction perpendicular to the direction of motion, and in the plane determined by the tangent to the trajectory formed by the center of gravity of a missile and the axis of the missile. (This plane is called the plane of yaw.)

drag. The component of the force of air resistance acting in a direction opposite to the motion of the center of gravity of a missile.

elevation. See *angle of departure.*

exterior ballistics. See *ballistics.*

horizontal range. The horizontal component of the distance from the beginning of the trajectory (point of origin) to any other point on the trajectory.

initial velocity. (muzzle velocity) The projectile velocity at the moment that the projectile ceases to be acted upon by propelling forces.

For a gun-fired projectile the initial velocity, expressed in feet or meters per second, is called *muzzle velocity* or *initial velocity.* It is obtained by measuring the velocity over a distance forward of the gun, and correcting back to the muzzle for the retardation in flight.

For a rocket-fired projectile, the velocity attained at the moment when rocket burnout occurs is the initial velocity. A slightly fictitious value is used, referred to an assumed point as origin of the trajectory.

The initial velocity of a bomb dropped from an airplane is the speed of the airplane.

Magnus force. The component of the force of air resistance acting in a direction perpendicular to the plane of yaw (see *crosswind force*). It is caused by the action between the boundary layer of air rotating with the projectile and the air stream. This force is small, compared with *drag* and *crosswind force.*

muzzle velocity. The velocity with which the projectile leaves the muzzle of the gun. See *initial velocity.*

slant range. The distance, measured along a straight line, from the beginning of the trajectory (point of origin) to a point on the trajectory.

stability factor. A factor which indicates the relative stability of a projectile under given conditions. It is dependent upon the force moments acting to align the projectile axis with the tangent to the trajectory, and the overturning force moments. For the projectile to be stable, the stability factor must be greater than unity.

standard atmosphere. An average or representative structure of the air used as a point of departure in computing firing tables for projec-

tiles. The standard atmosphere adopted in 1956 for the Armed Services and now known as the U. S. Standard Atmosphere, is that used by the International Civil Aviation Organization (ICAO). This Standard Atmosphere assumes a ground pressure of 760 m/m of mercury, a ground temperature of 15° C, air density at ground level of 0.076,475 lb/ft^3, and a temperature gradient to the beginning of the stratosphere expressed by the formula

absolute temperature $T(°K) = 288.16\text{-}6.5H$

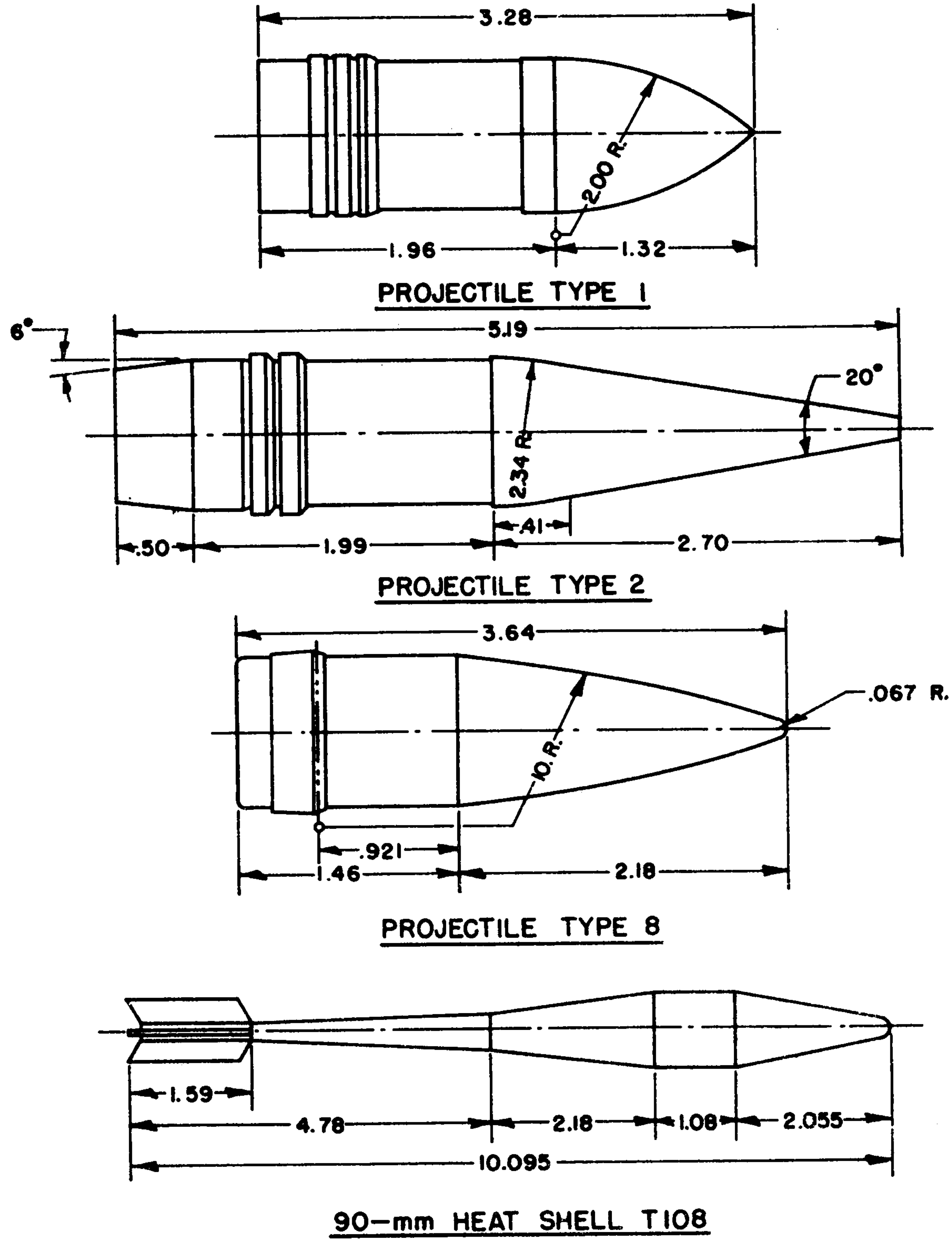
where H is height above sea level in kilometers.

In the stratosphere (above 11 kilometers) the temperature is assumed constant at 216.66° K. For firing table purposes wind velocities of zero are assumed.

superelevation. An added positive angle in antiaircraft gunnery that compensates for the fall of the projectile during the time of flight due to the pull of gravity; the angle the gun or projector must be elevated above the gun-target line.

trajectory. The path followed in space by the center of gravity of a missile while in free flight.

yaw. The angle between the direction of motion of a projectile and the axis of the projectile, referred to either as *yaw,* or more completely, *angle of yaw.* The angle of yaw increases with time of flight in an unstable projectile (see *stability factor*) and decreases to a constant value, called the *yaw of repose,* or the *repose angle of yaw,* in a stable projectile.



ALL DIMENSIONS IN CALIBERS

Figure 1. Projectile Shapes

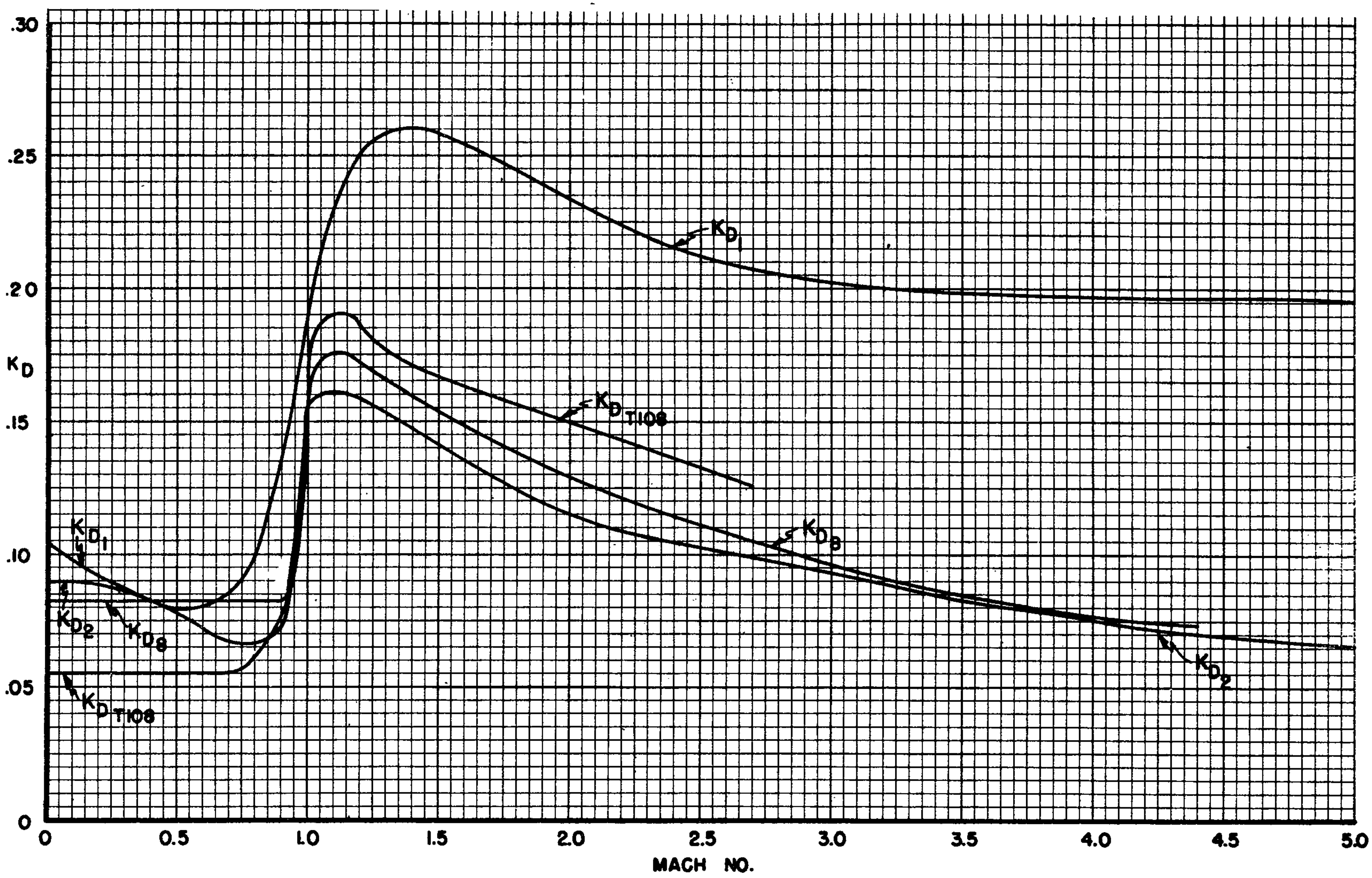


Figure 2. Drag Coefficient vs Mach Number

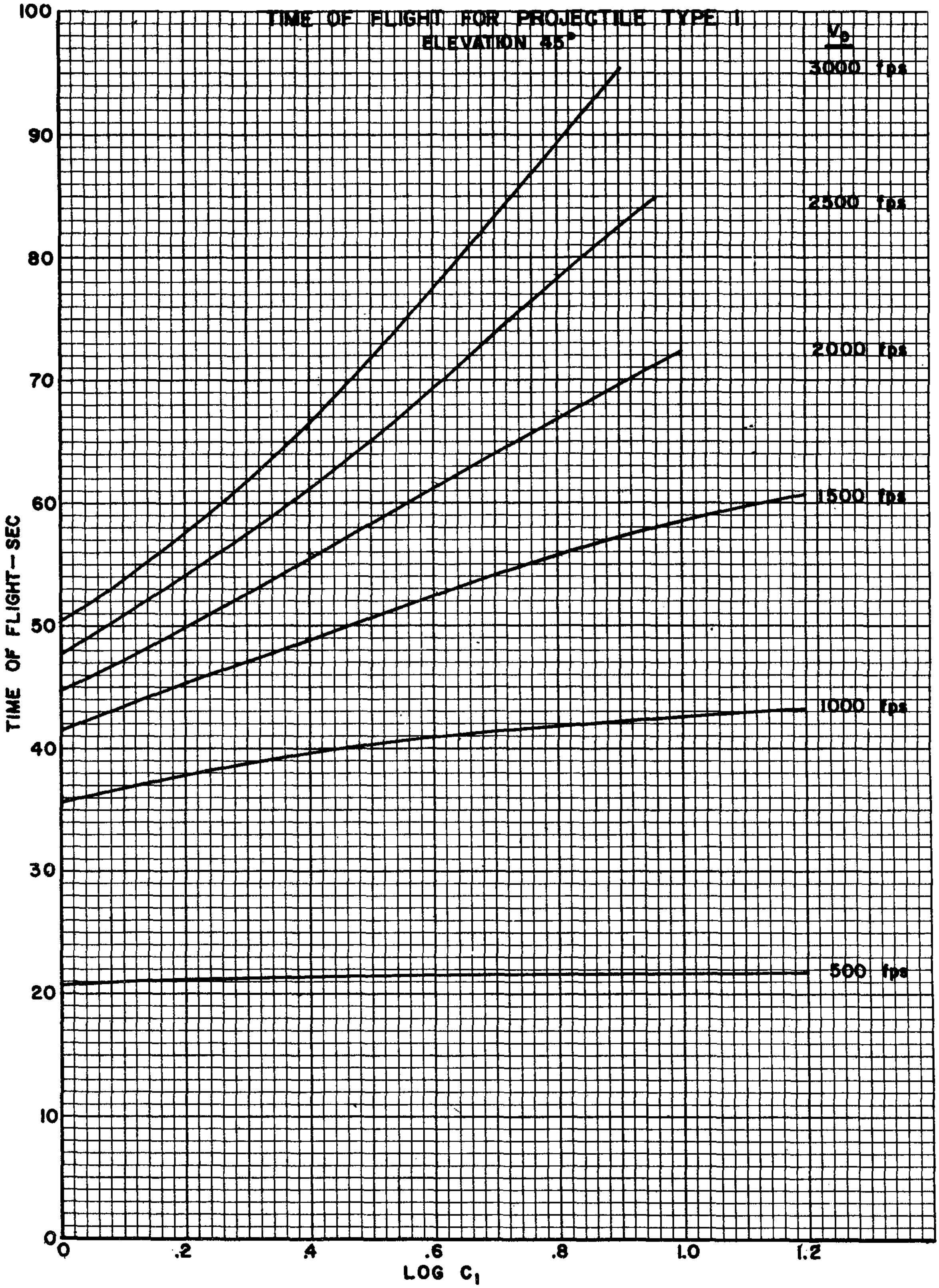


Figure 3. Time of Flight for Projectile Type 1

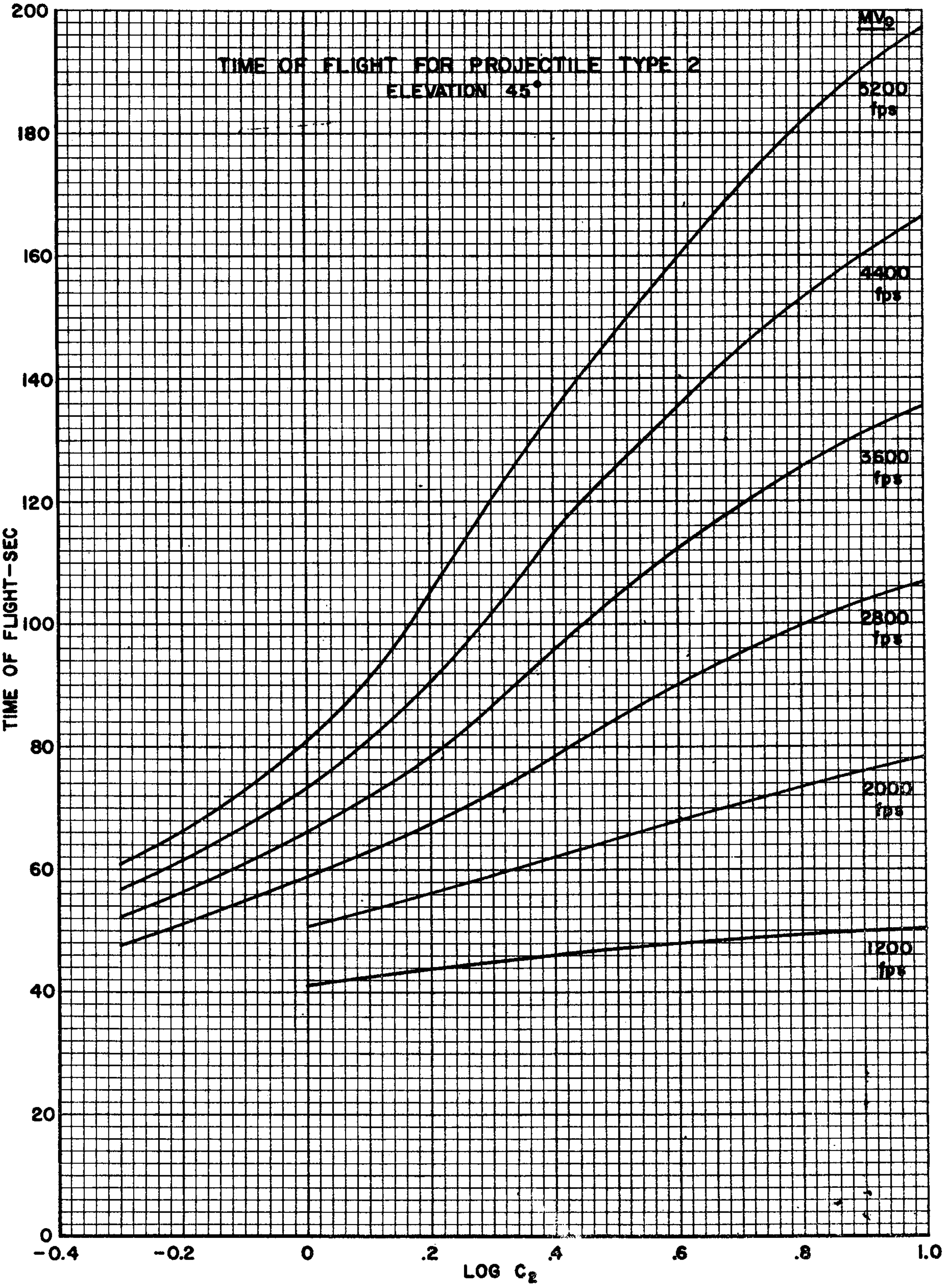


Figure 4. Time of Flight for Projectile Type 2

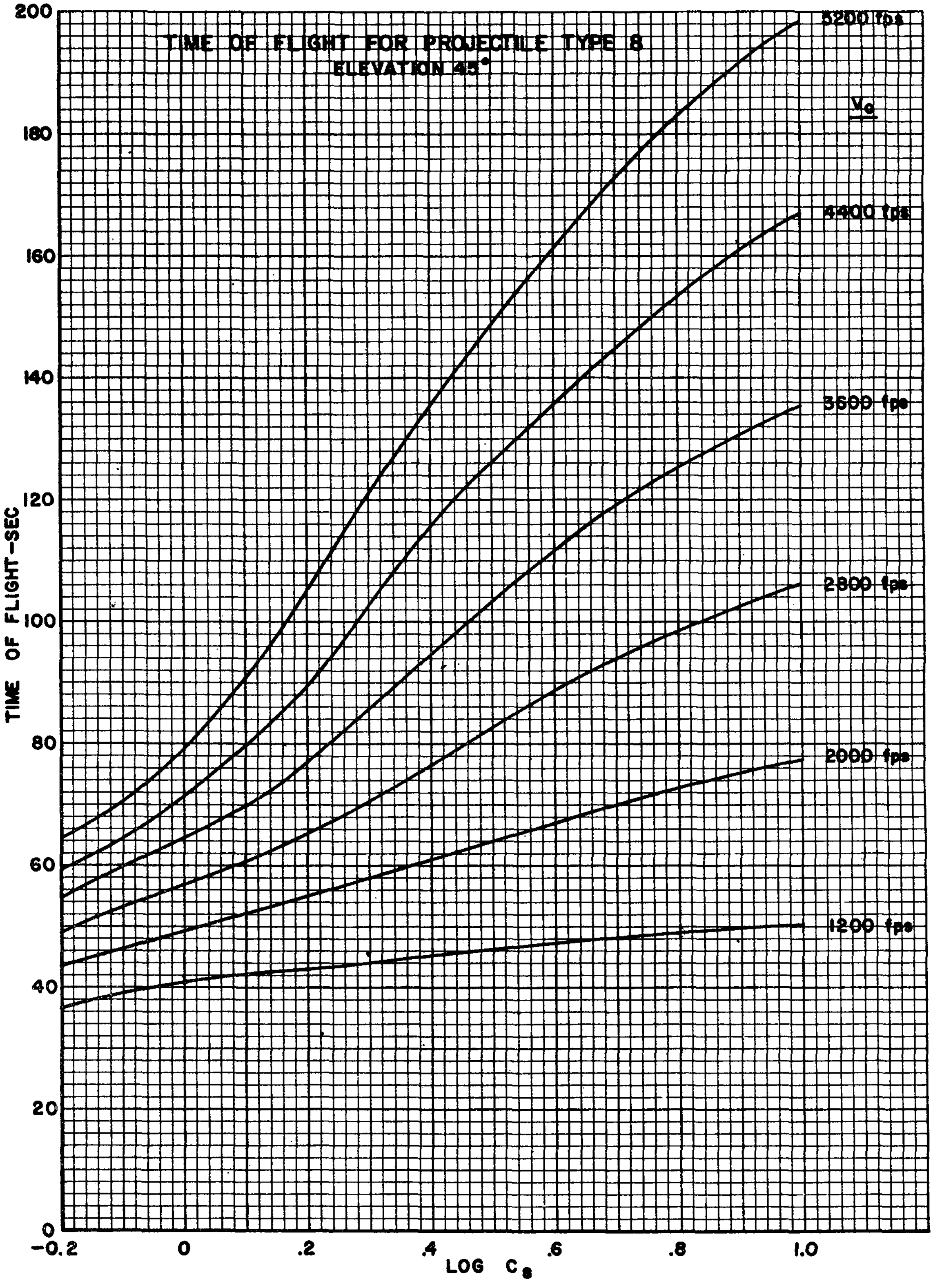


Figure 5. Time of Flight for Projectile Type 8

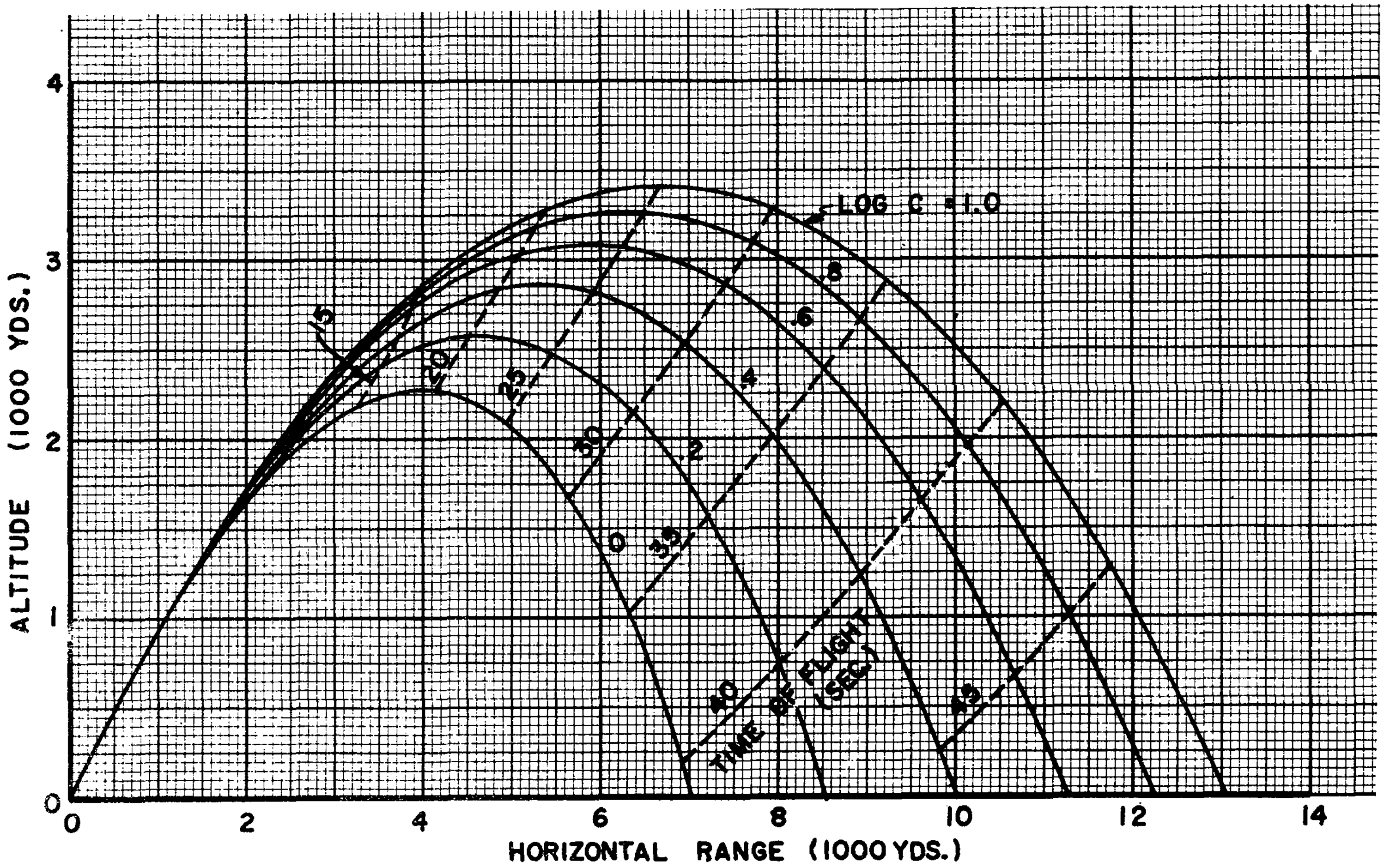


Figure 6 (1 of 12). Trajectories for Projectile Type 2, 45° elevation, 1200 fps muzzle velocity

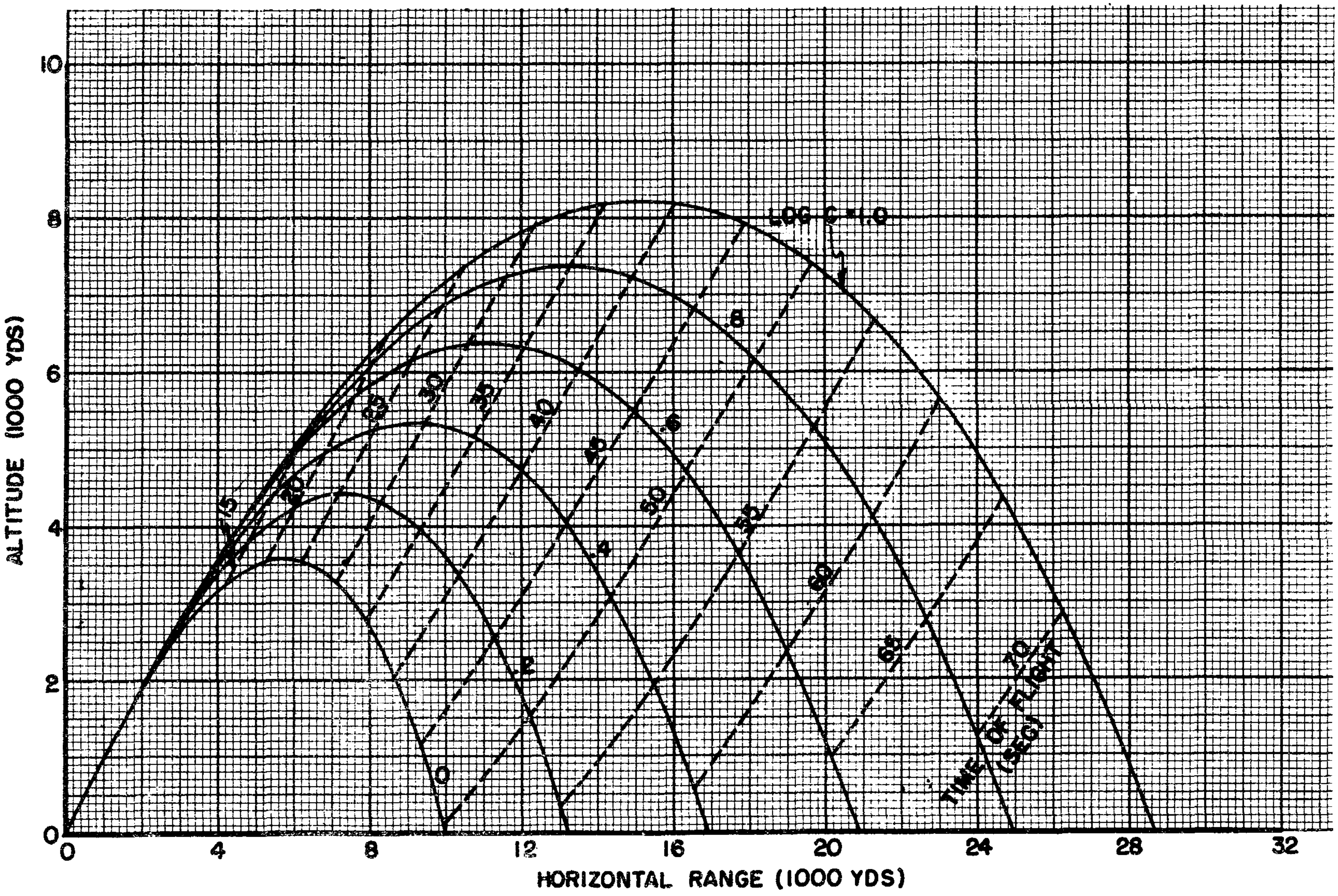


Figure 6 (2 of 12). Trajectories for Projectile Type 2, 45° elevation, 2000 fps muzzle velocity

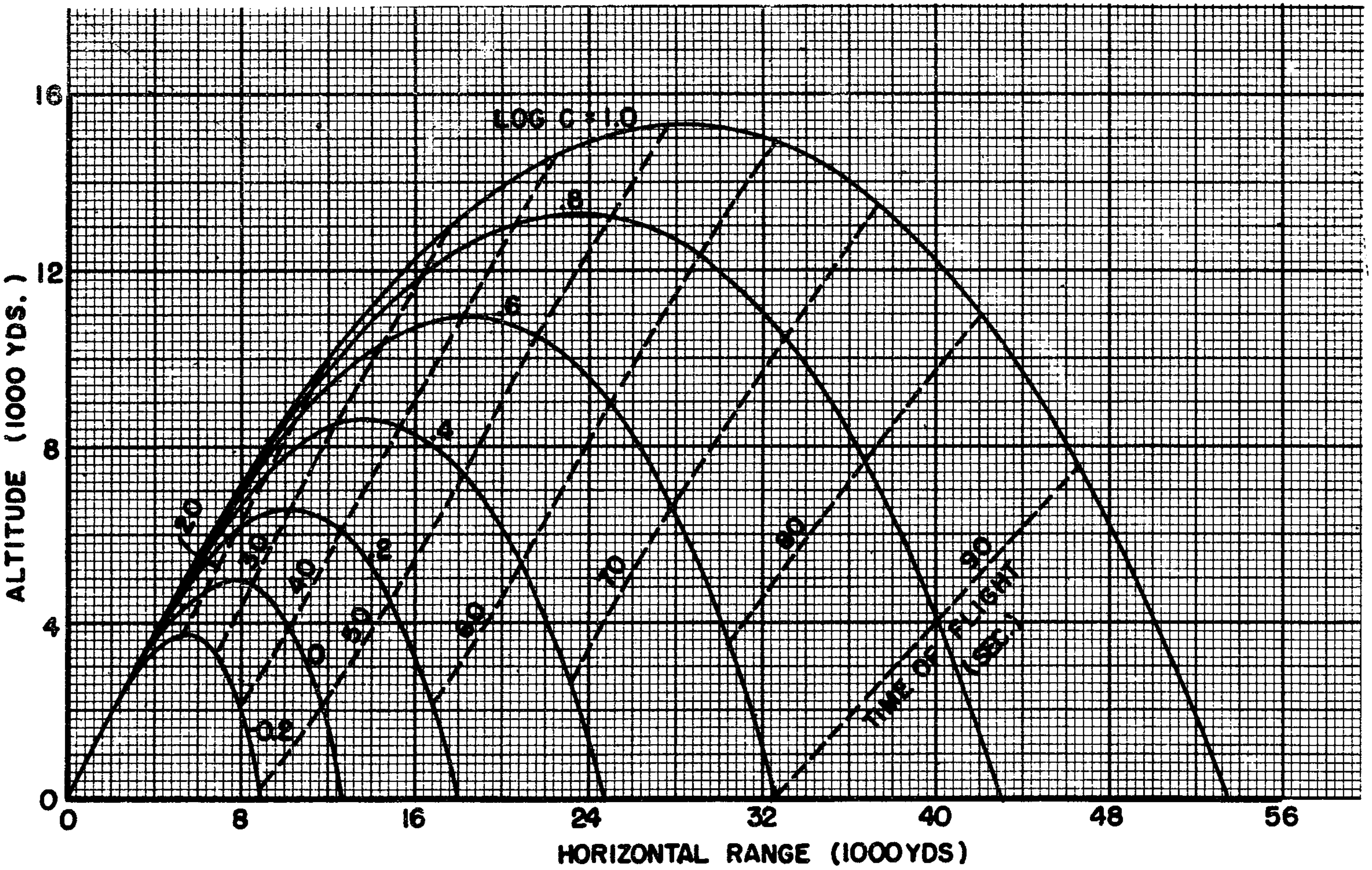


Figure 6 (3 of 12). Trajectories for Projectile Type 2, 45° elevation, 2800 fps muzzle velocity

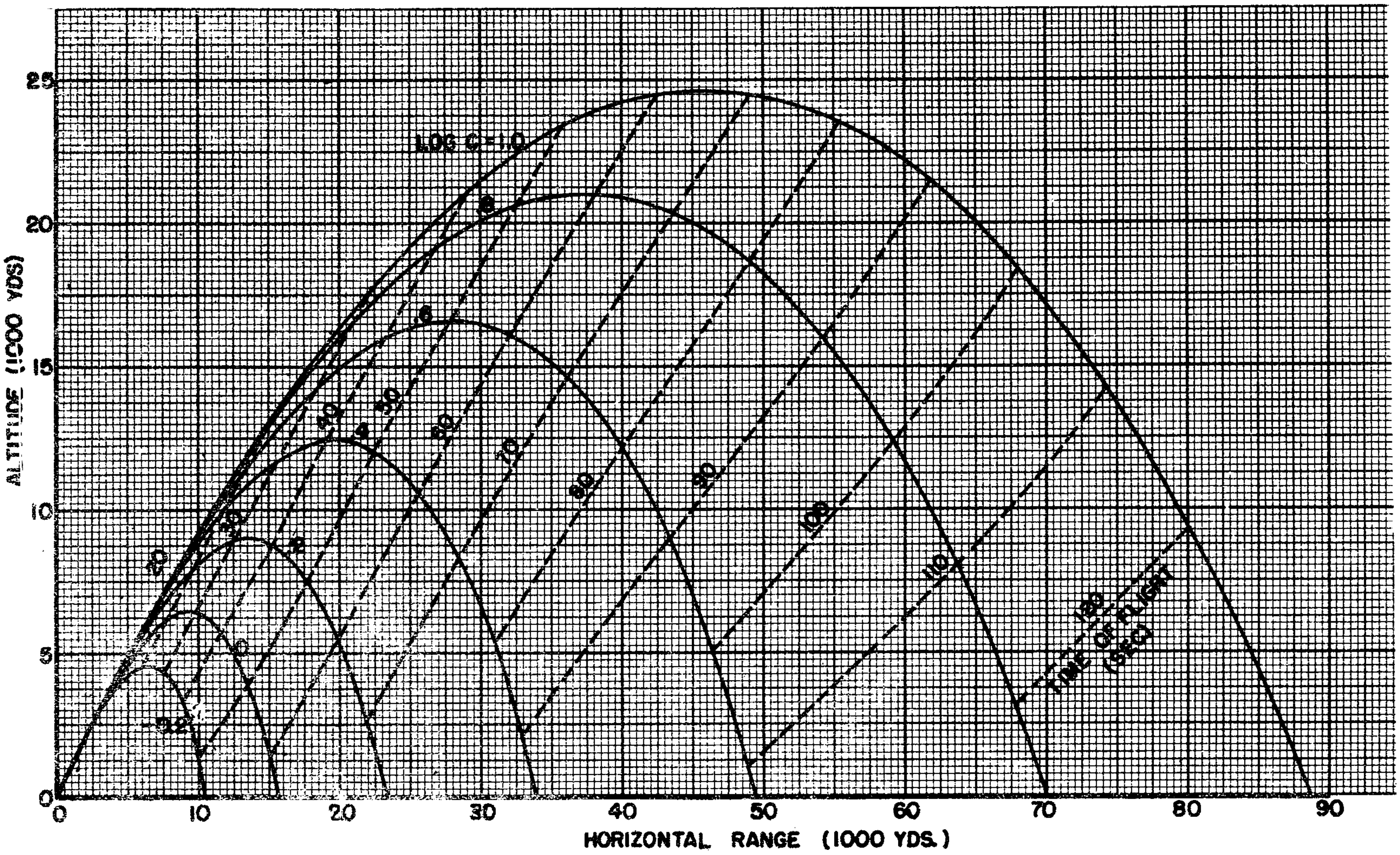


Figure 6 (4 of 12). Trajectories for Projectile Type 2, 45° elevation, 3600 fps muzzle velocity

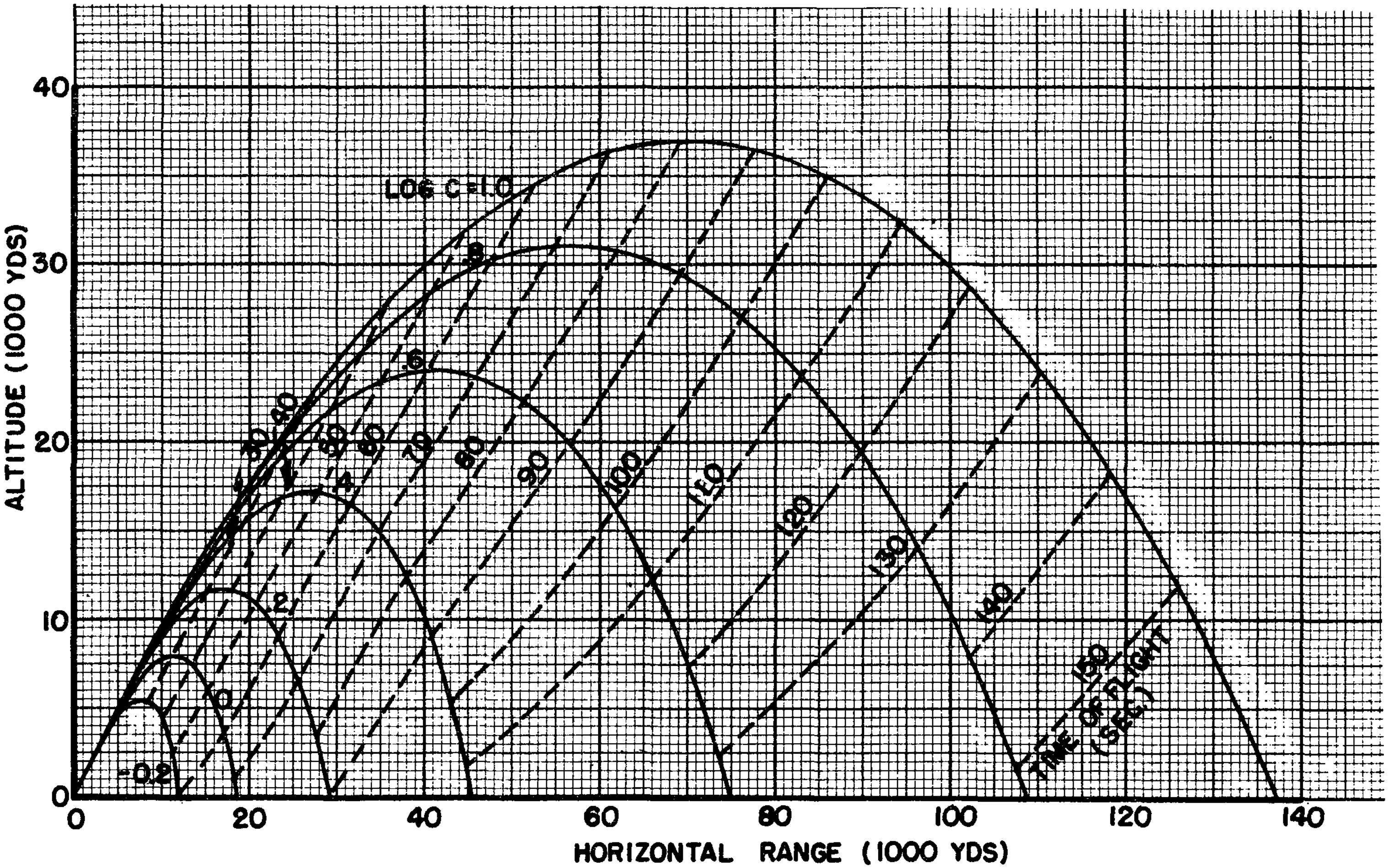


Figure 6 (5 of 12). Trajectories for Projectile Type 2, 45° elevation, 4400 fps muzzle velocity

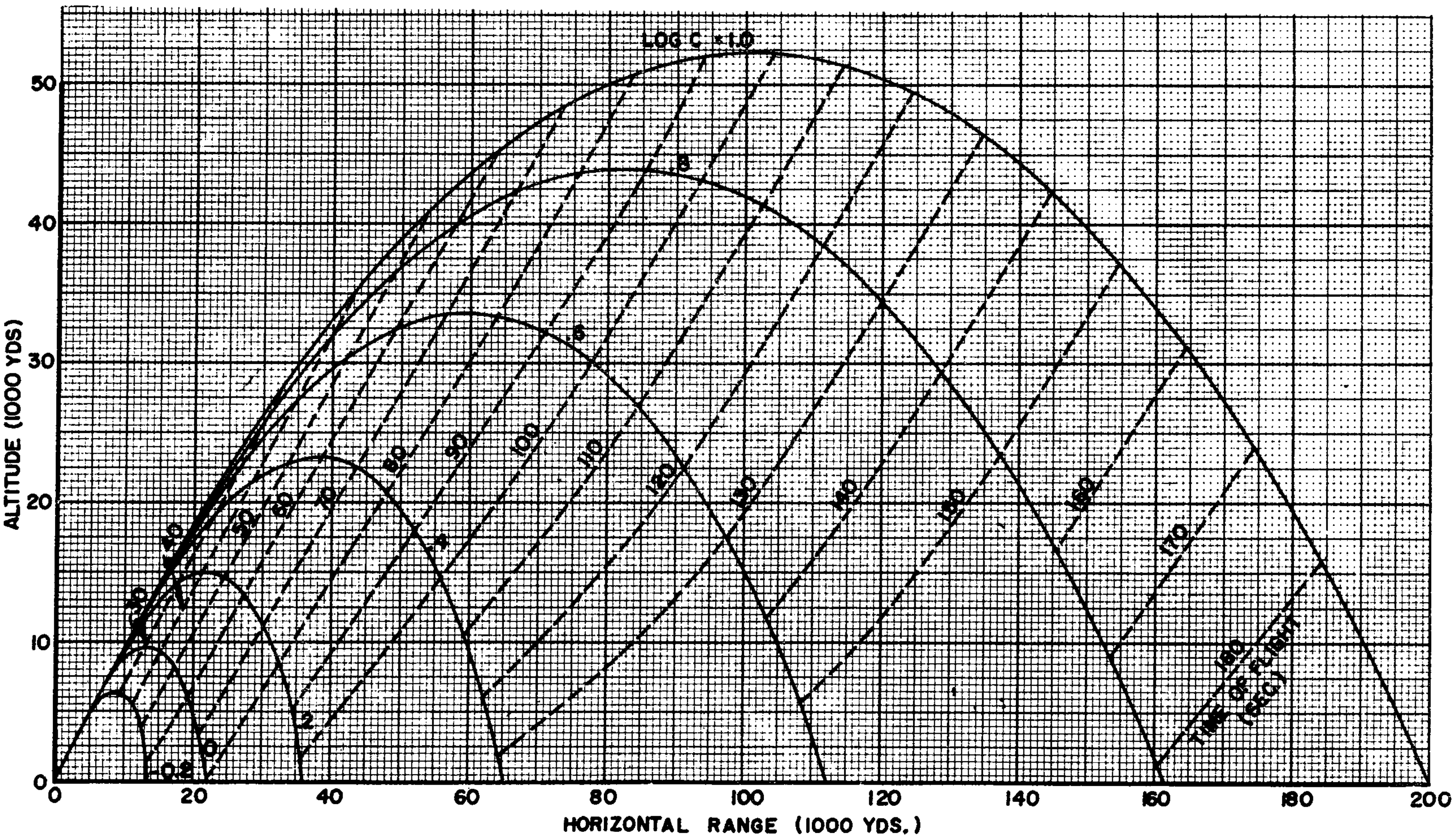


Figure 6 (6 of 12). Trajectories for Projectile Type 2, 45° elevation, 5200 fps muzzle velocity

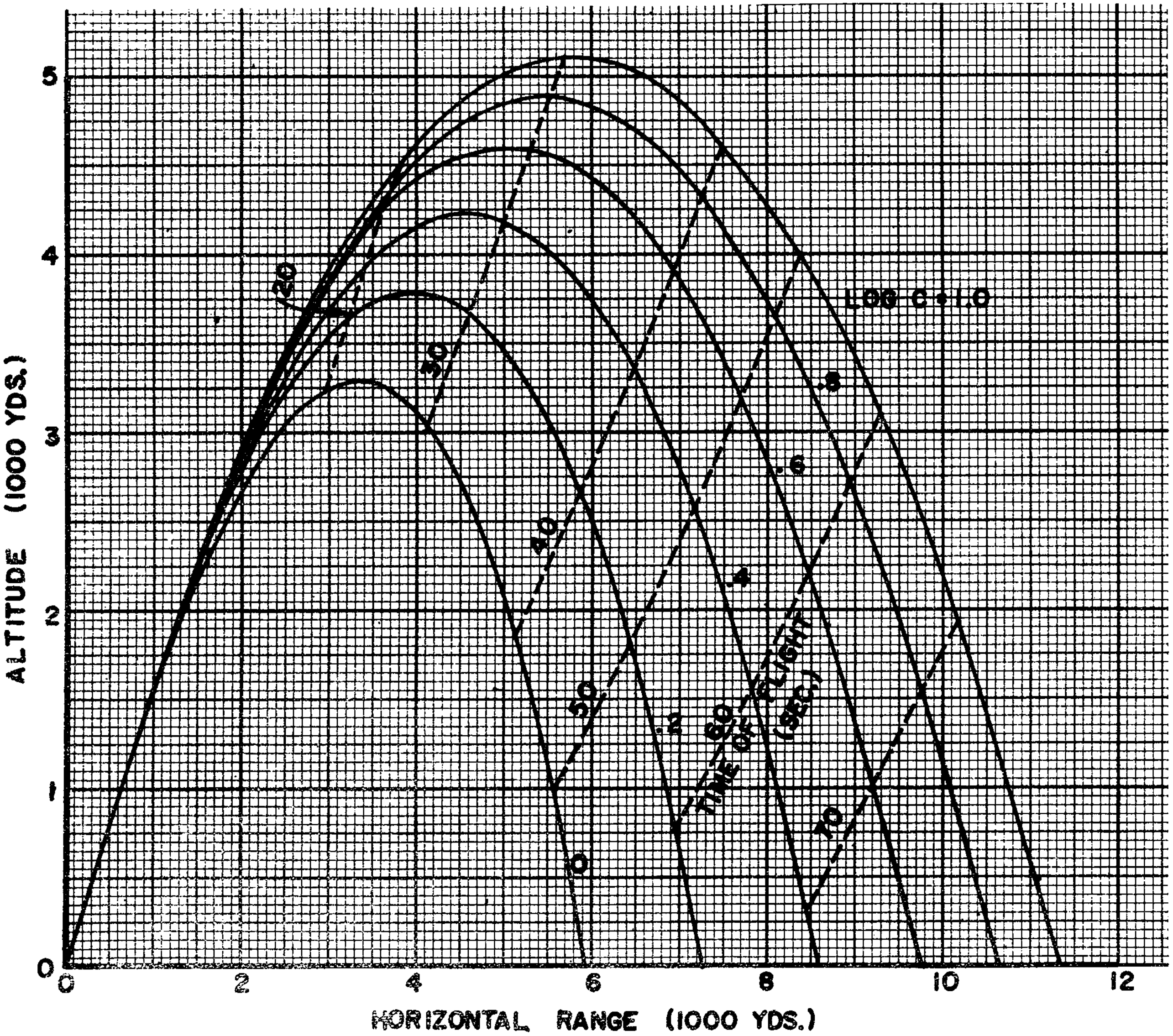


Figure 6 (7 of 12). Trajectories for Projectile Type 2, 60° elevation, 1200 fps muzzle velocity

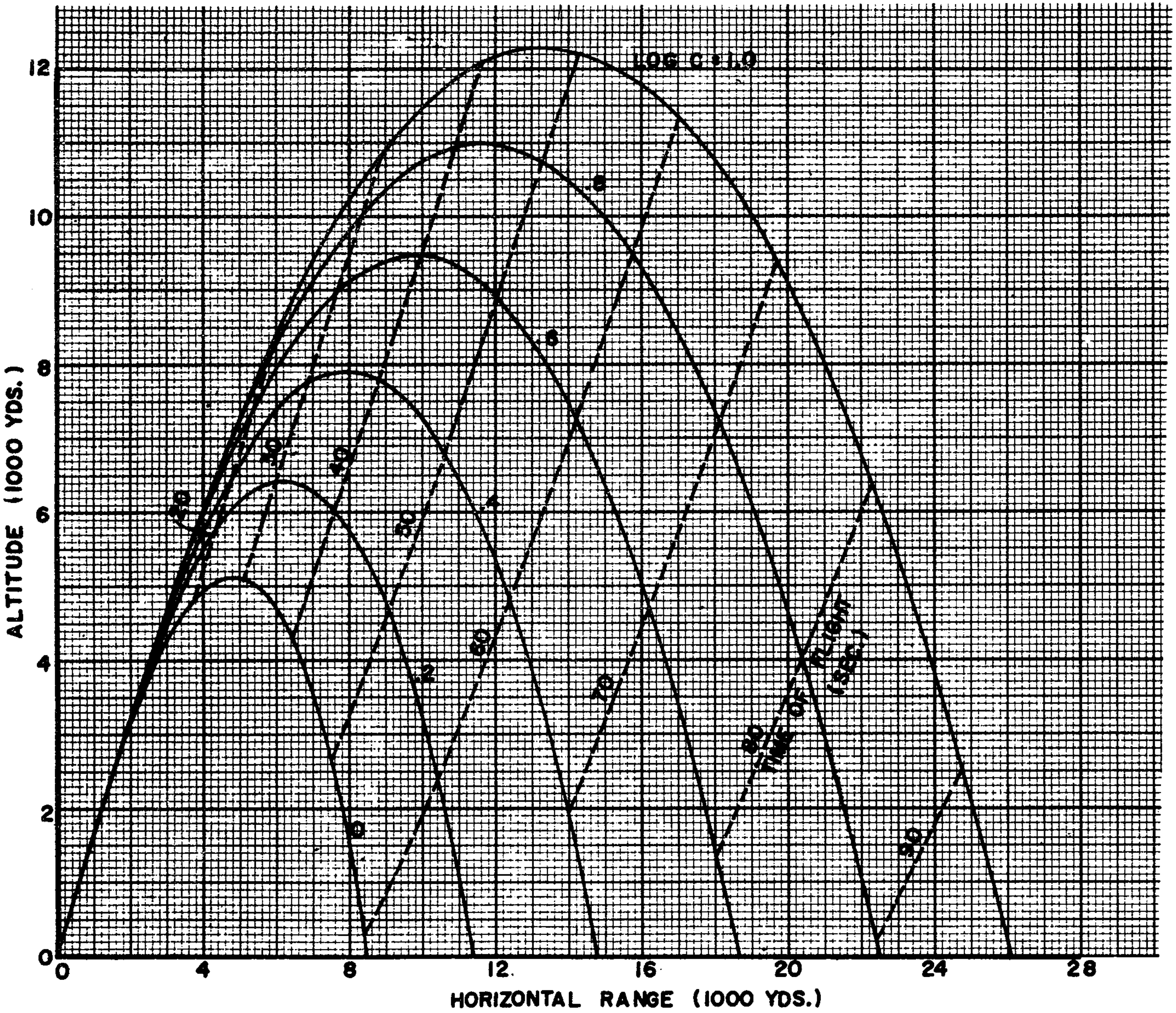


Figure 6 (8 of 12). Trajectories for Projectile Type 2, 60° elevation, 2000 fps muzzle velocity

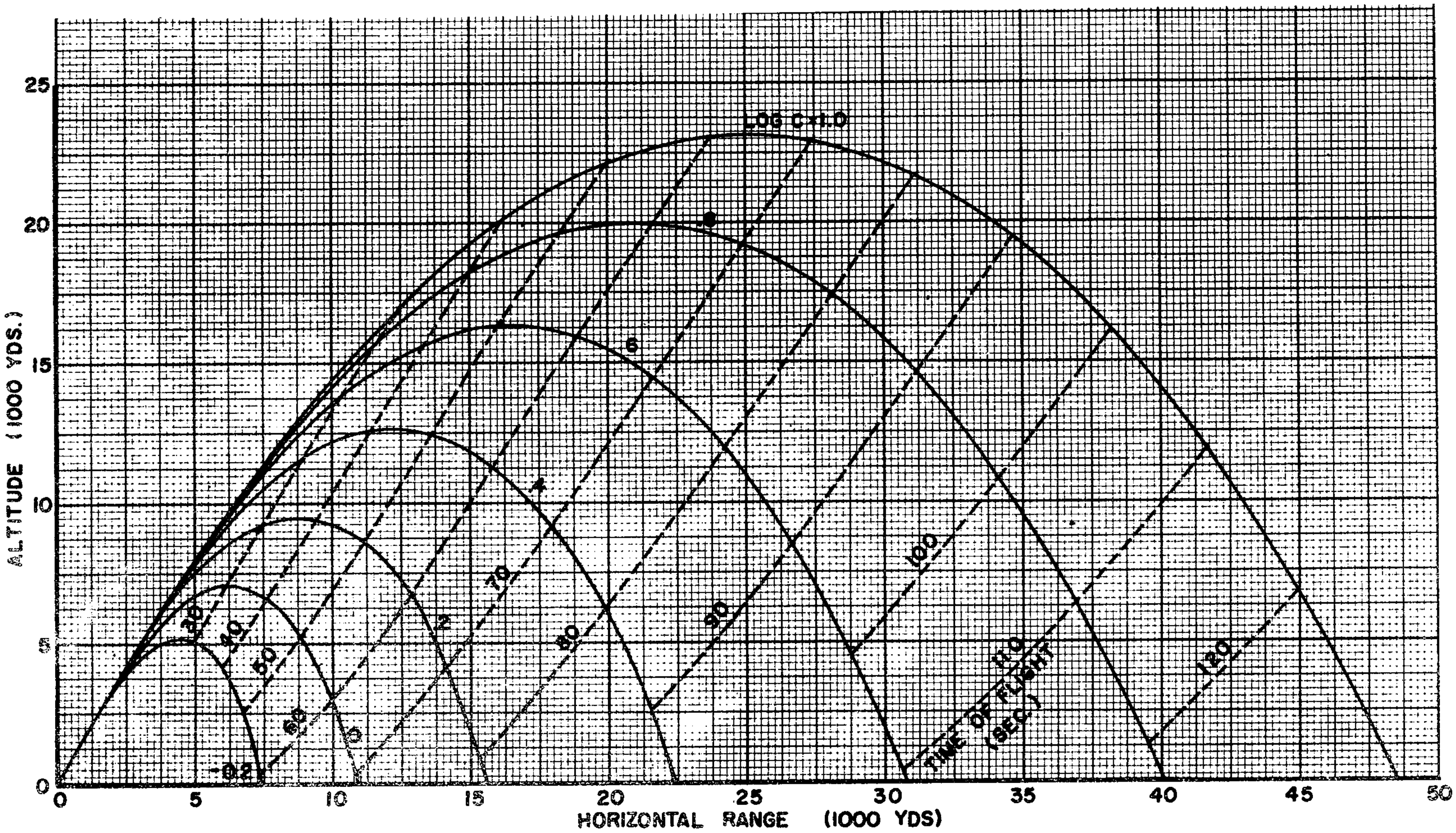


Figure 6 (9 of 12). Trajectories for Projectile Type 2, 60° elevation, 2800 fps muzzle velocity

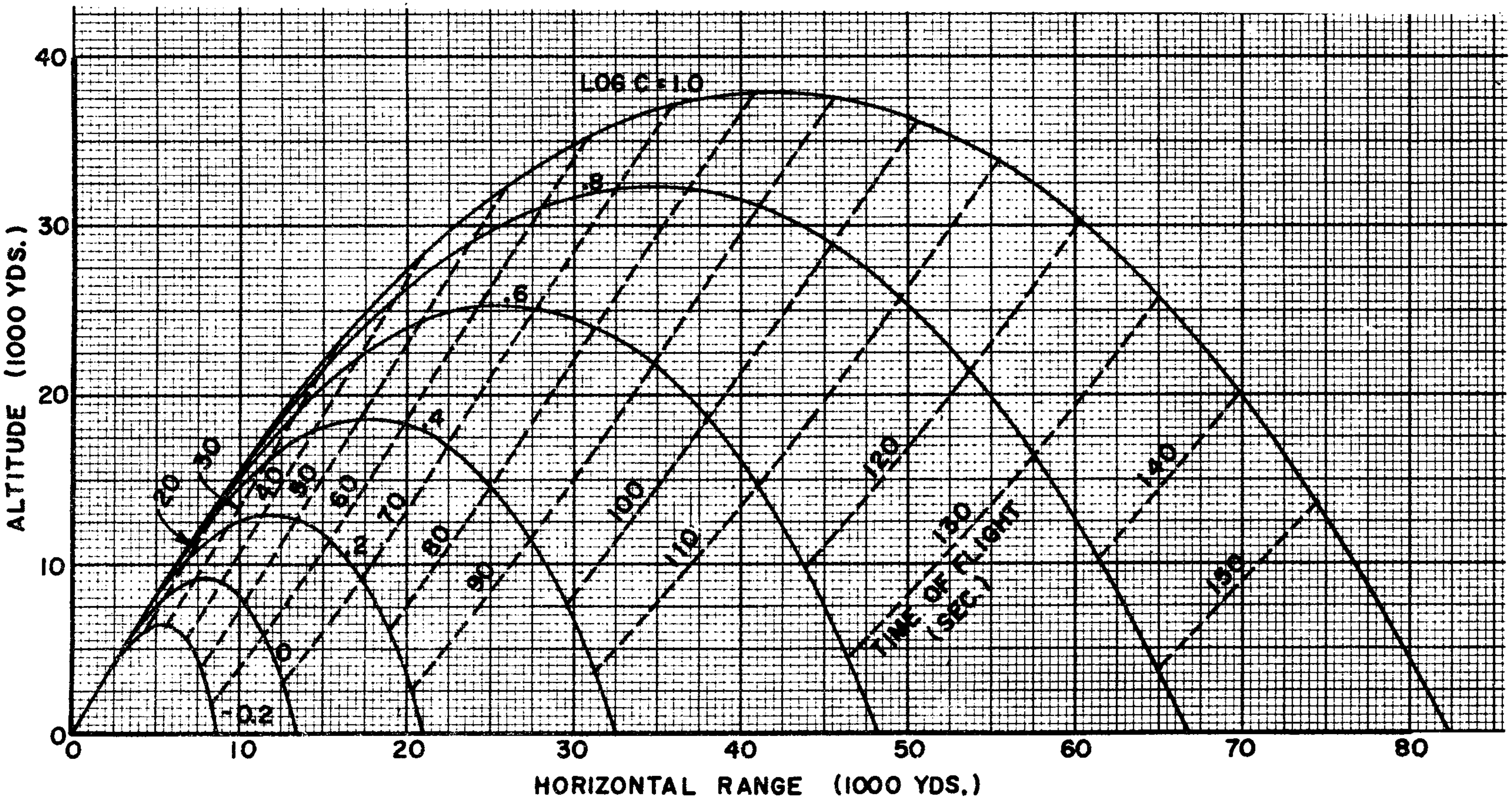


Figure 6 (10 of 12). Trajectories for Projectile Type 2, 60° elevation, 3600 fps muzzle velocity

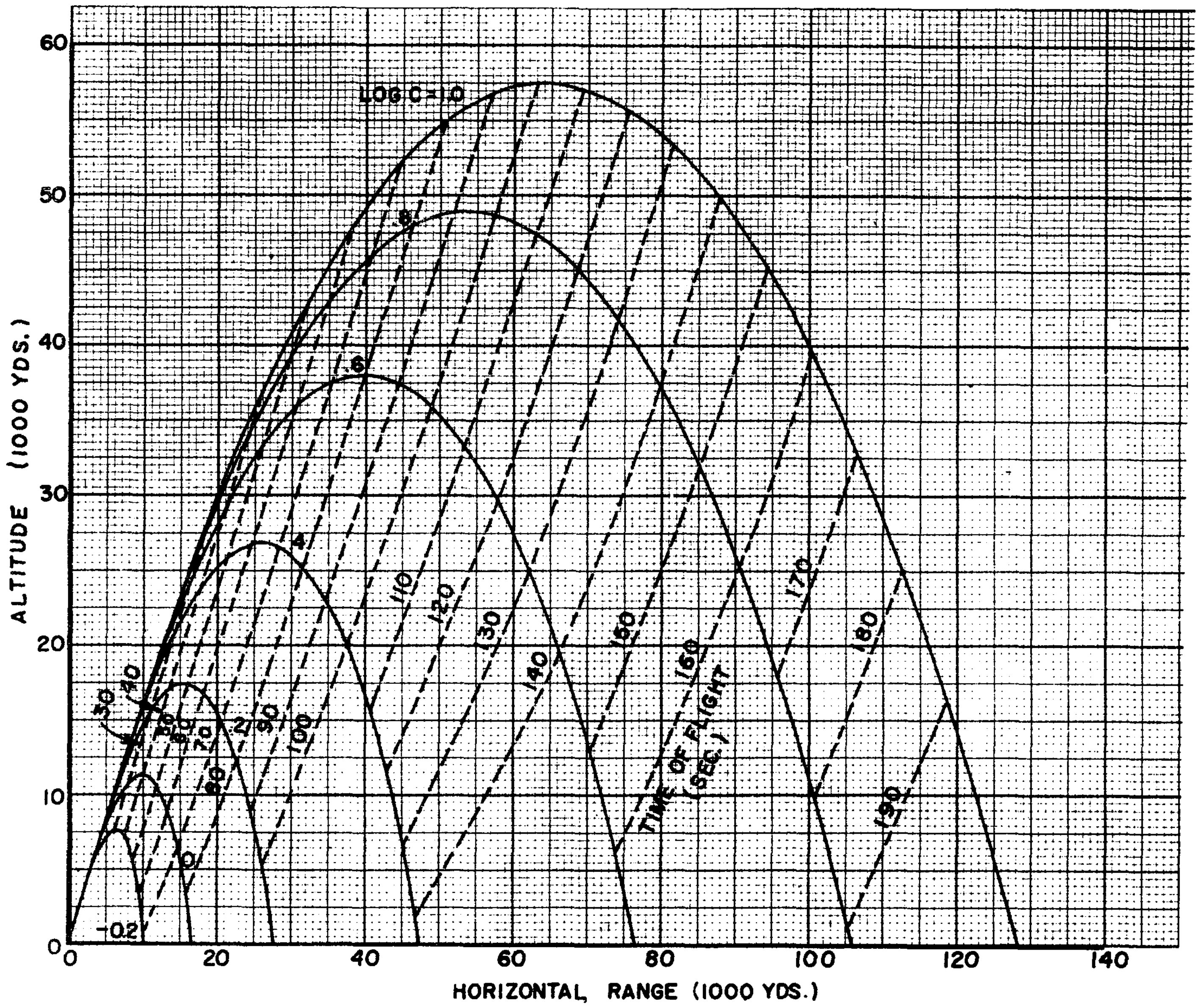


Figure 6 (11 of 12). Trajectories for Projectile Type 2, 60° elevation, 4400 fps muzzle velocity

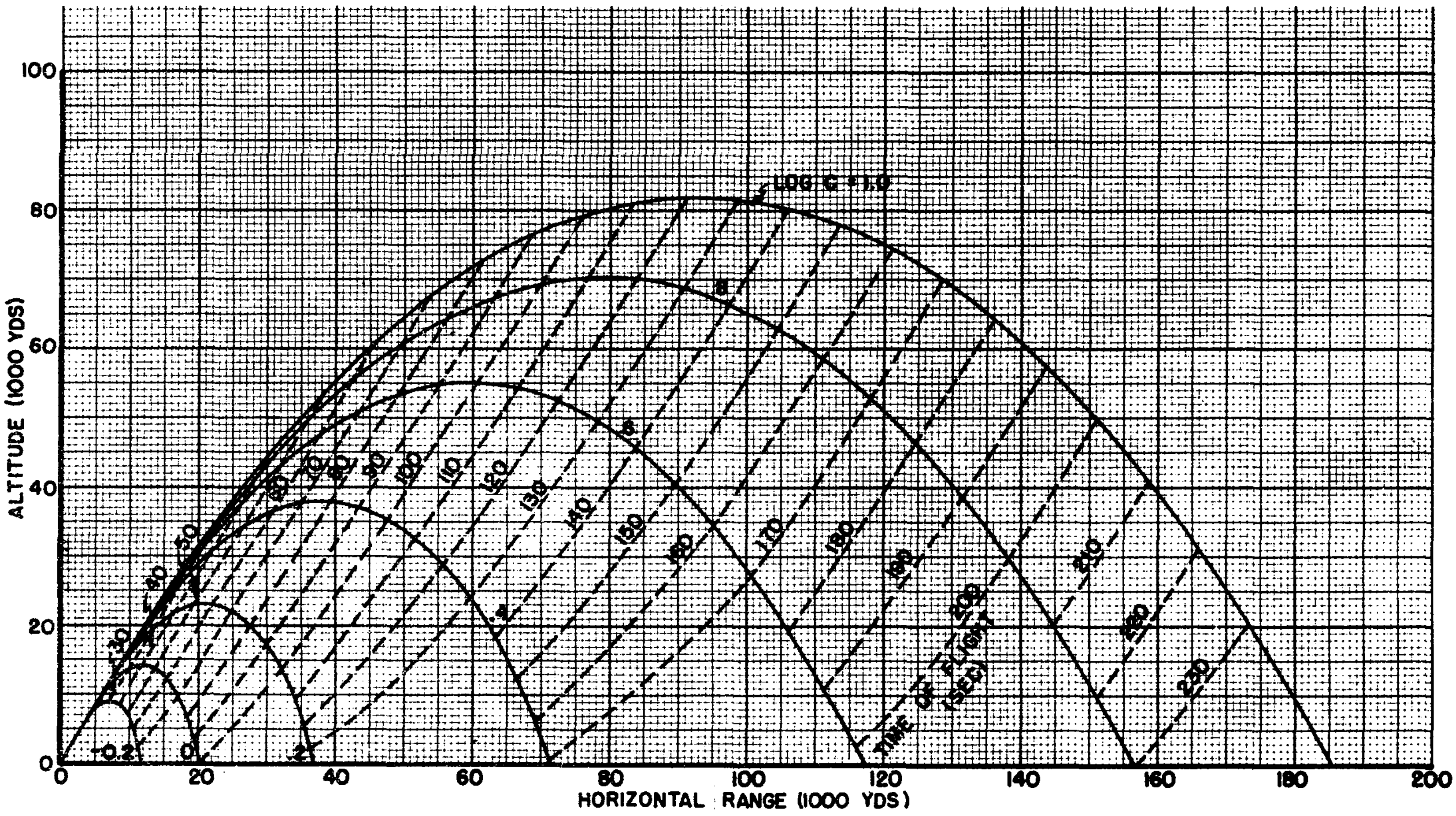


Figure 6 (12 of 12). Trajectories for Projectile Type 2, 60° elevation, 5200 fps muzzle velocity